Mat-C.1 harj2

21.3. 2012

Alustuksia

1.

a)

> $f := 1 + \frac{\sin(x)}{1+x^2}$

$$f := 1 + \frac{\sin(x)}{1+x^2} \quad (2.1)$$

> $subs(x=-2.0, f); evalf(\%)$ *# Sijoita x:n paikalle -2.0 lausekkeessa f.*

$$1 + 0.2000000000 \sin(-2.0)$$

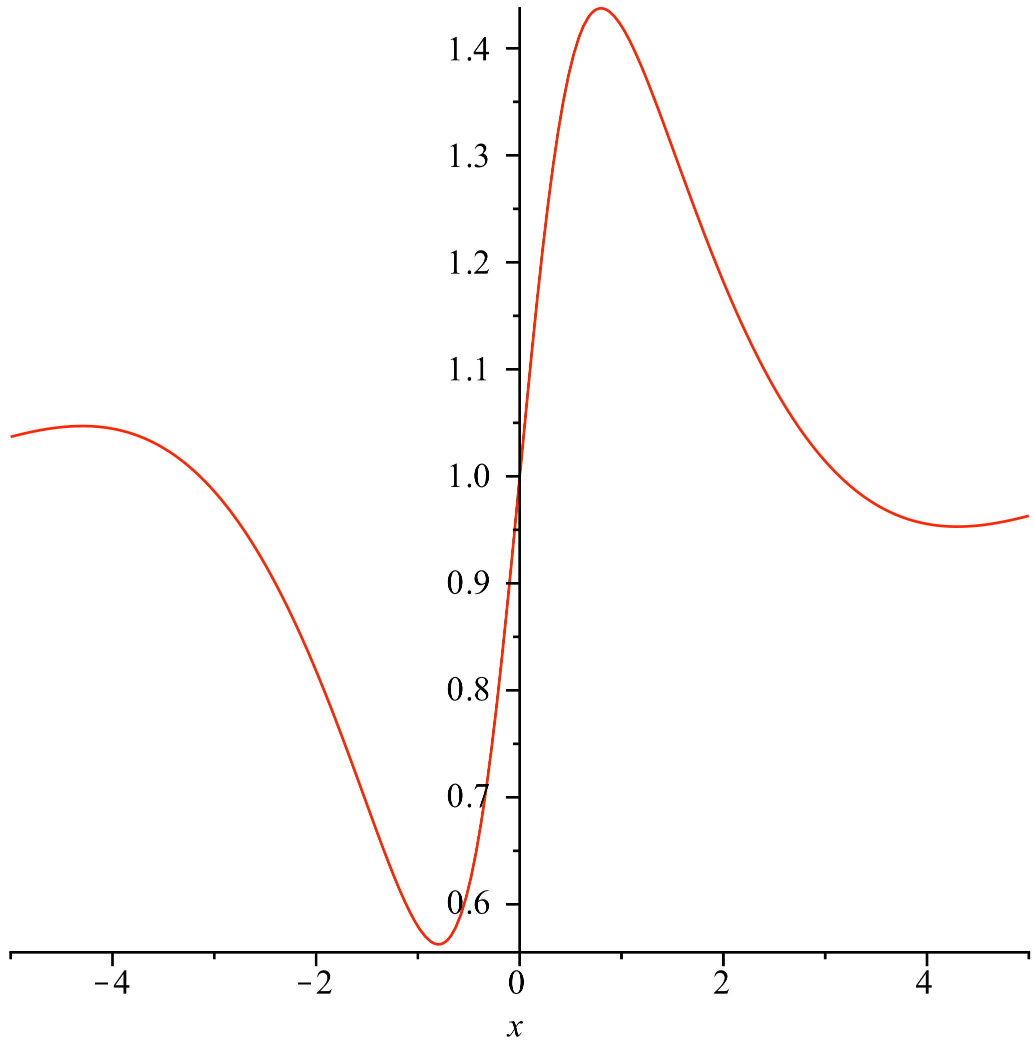
$$0.8181405146 \quad (2.2)$$

> $eval(f, x=-2.0)$ *# Evaluoi f, ehdolla x=−2.0*

$$0.8181405146 \quad (2.3)$$

>

> $plot(f, x=-5..5);$



b) M"a"aritell"a"an f funktioksi:

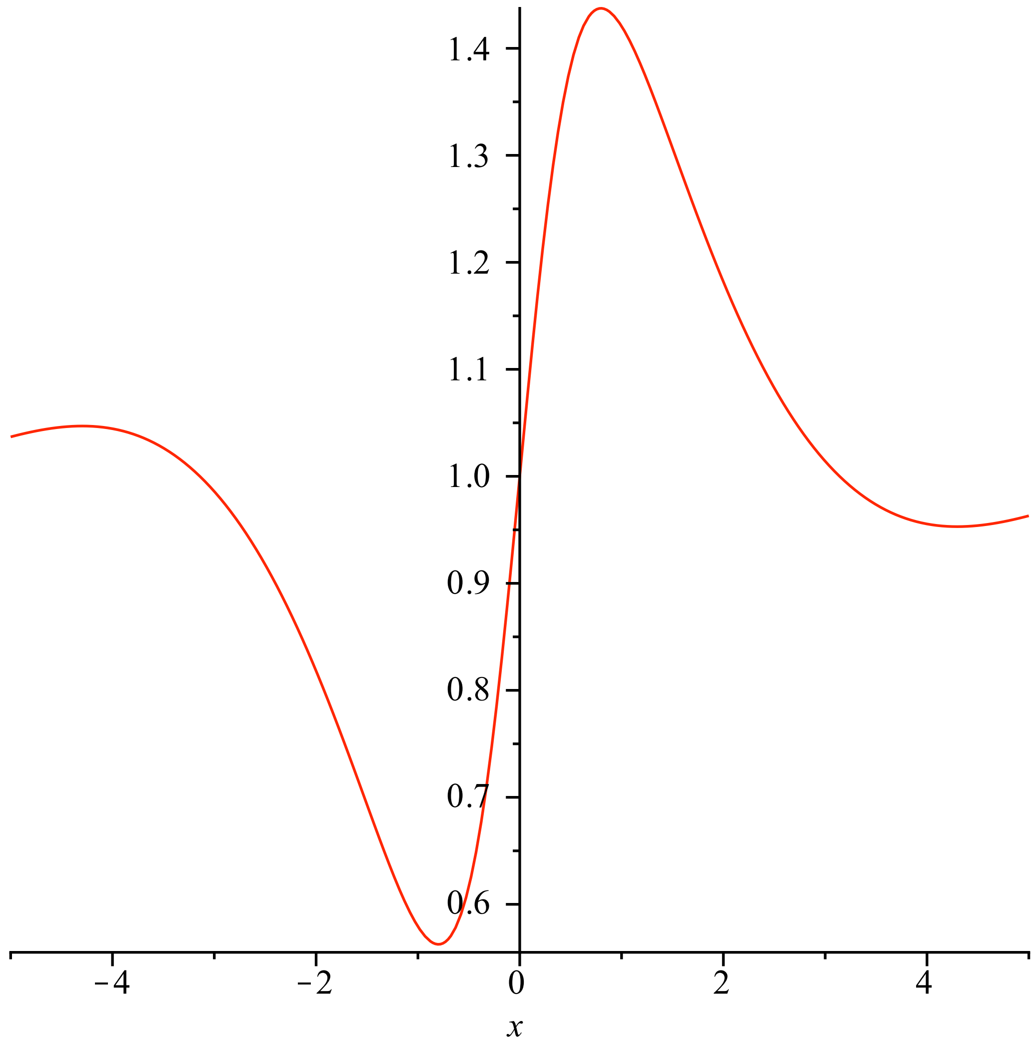
> $f := x \rightarrow 1 + \frac{\sin(x)}{1+x^2}$

$$f := x \rightarrow 1 + \frac{\sin(x)}{1+x^2} \quad \textbf{(2.4)}$$

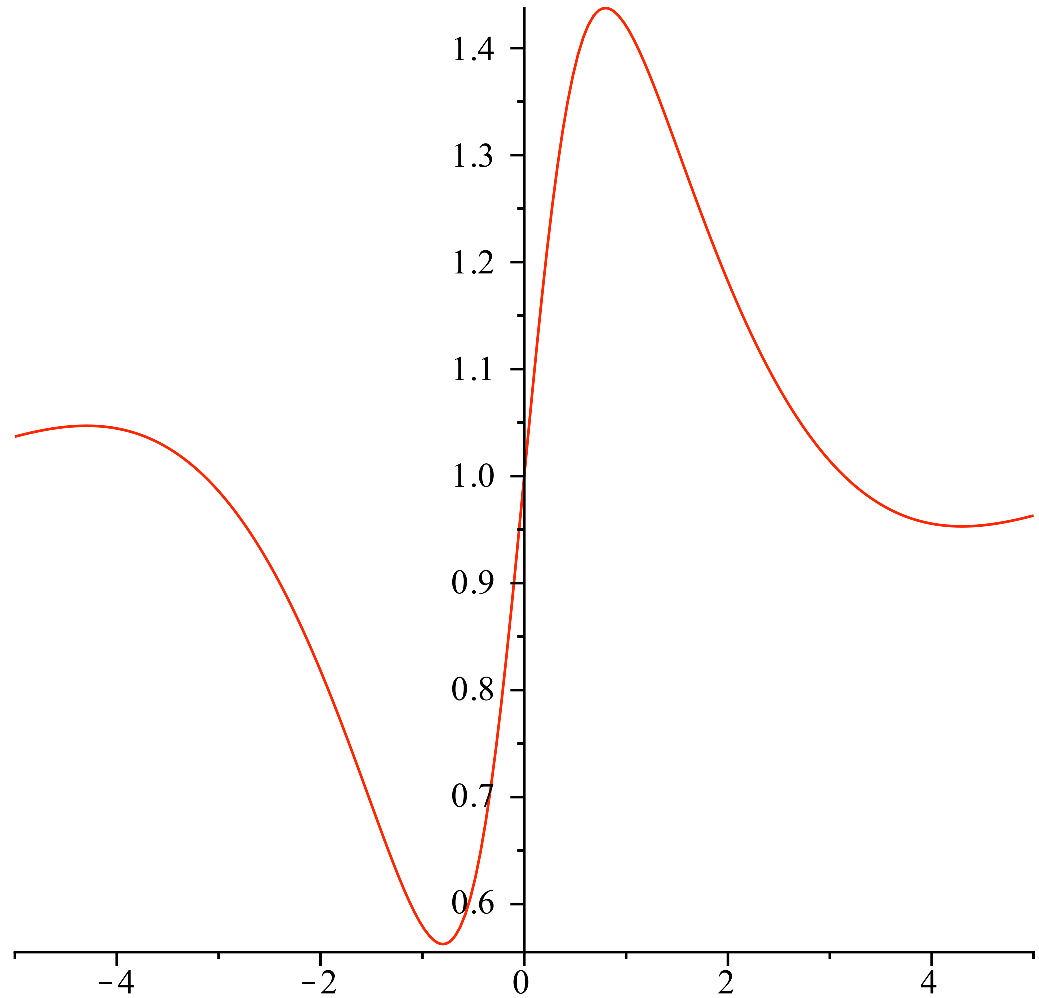
> $f(-2.0)$

$$0.8181405146 \quad \textbf{(2.5)}$$

> $plot(f(x), x=-5..5)$



> *plot*(*f*, −5 ..5)




```
> read("/Users/heikki/opetus/peruskurssi/v2-3/maple/v202.mpl");
> print(linspace)
proc( )                                                        (1)
    local i, n, a, b;
    a := args[1];
    b := args[2];
    if nargs = 2 then n := 10 else n := args[3] end if;
    [seq(a + i * (b − a) / (n − 1), i = 0 ..n − 1)]
end proc
```

4. (Osder, Laplacen DY, harmoniset fkt.)

a)

```
> LapDy := diff(u(x, y), x, x) + diff(u(x, y), y, y) = 0
```

$$LapDy := \frac{\partial^2}{\partial x^2} u(x, y) + \frac{\partial^2}{\partial y^2} u(x, y) = 0 \quad (3.1)$$

> $u := \arctan\left(\frac{y}{x}\right)$

$$u := \arctan\left(\frac{y}{x}\right) \quad (3.2)$$

> $subs(u(x,y) = u, LapDy);$

$$\frac{\partial^2}{\partial x^2}\arctan\left(\frac{y}{x}\right) + \frac{\partial^2}{\partial y^2}\arctan\left(\frac{y}{x}\right) = 0 \quad (3.3)$$

> $eval(\%);$

$$\frac{2\,y}{x^3\left(1+\frac{y^2}{x^2}\right)} - \frac{2\,y^3}{x^5\left(1+\frac{y^2}{x^2}\right)^2} - \frac{2\,y}{x^3\left(1+\frac{y^2}{x^2}\right)^2} = 0 \quad (3.4)$$

```
> simplify(%);
```

$$0 = 0 \quad (3.5)$$

b)

> *restart* :

> $CR1 := \frac{\partial}{\partial x}u(x,y) = \frac{\partial}{\partial y}v(x,y);$

$$CR1 := \frac{\partial}{\partial x}\,u(x,y) = \frac{\partial}{\partial y}\,v(x,y) \quad (3.6)$$

> $CR2 := \frac{\partial}{\partial y}u(x,y) = -\frac{\partial}{\partial x}v(x,y);$

$$CR2 := \frac{\partial}{\partial y}\,u(x,y) = -\left(\frac{\partial}{\partial x}\,v(x,y)\right) \quad (3.7)$$

> $diff\sim(CR1, x)$

$$\frac{\partial^2}{\partial x^2}\,u(x,y) = \frac{\partial^2}{\partial y\,\partial x}\,v(x,y) \quad (3.8)$$

> $diff\sim(CR2, y)$

$$\frac{\partial^2}{\partial y^2}\,u(x,y) = -\left(\frac{\partial^2}{\partial y\,\partial x}\,v(x,y)\right) \quad (3.9)$$

> **(3.8)** $+\sim$ **(3.9)**;

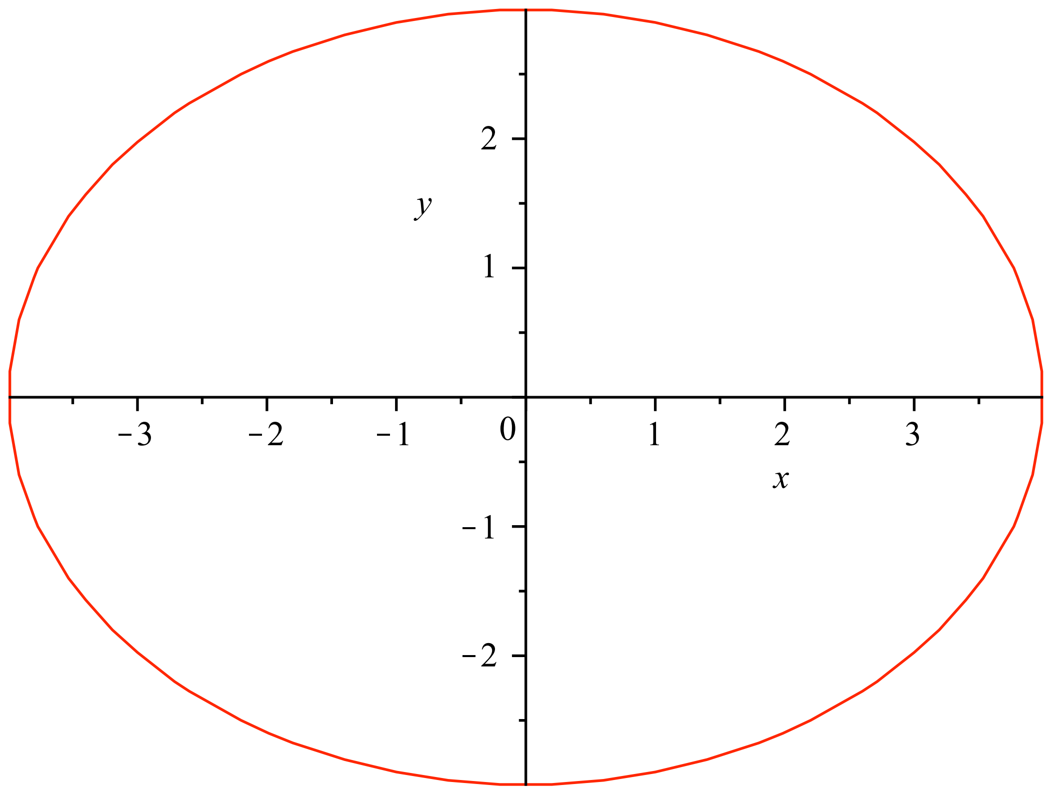
$$\frac{\partial^2}{\partial x^2}\,u(x,y) + \frac{\partial^2}{\partial y^2}\,u(x,y) = 0 \quad (3.10)$$

▶ Jonot ja listat

▼ 3. DokuT

Ellipsin $9 \cdot x^2 + 16 \cdot y^2 = 144$ sisään piirrettävä suorakulmio, jonka ala = max.

```
> restart:
> ellipsi := 9·x^2 + 16·y^2 = 144
```
$$ellipsi := 9\,x^2 + 16\,y^2 = 144 \tag{5.1}$$
```
> A := 4·x·y
```
$$A := 4\,x\,y \tag{5.2}$$
```
> Y := solve(ellipsi, y);
```
$$Y := \frac{3}{4}\sqrt{-x^2+16},\ -\frac{3}{4}\sqrt{-x^2+16} \tag{5.3}$$
```
> y := Y[1]
```
$$y := \frac{3}{4}\sqrt{-x^2+16} \tag{5.4}$$
```
> A;  # y:n arvo sijoittui A:n lausekkeeseen (Muista Sokrates!)
```
$$3\,x\sqrt{-x^2+16} \tag{5.5}$$
```
> dA := diff(A, x);
```
$$dA := 3\sqrt{-x^2+16} - \frac{3\,x^2}{\sqrt{-x^2+16}} \tag{5.6}$$
```
> simplify(%);
```
$$-\frac{6\,(x^2-8)}{\sqrt{-x^2+16}} \tag{5.7}$$
```
> solve(% = 0, x);
```
$$-2\sqrt{2},\ 2\sqrt{2} \tag{5.8}$$
```
> x0 := max(%);
```
$$x0 := 2\sqrt{2} \tag{5.9}$$
```
> subs(x = x0, A);
```
$$6\sqrt{2}\,\sqrt{8} \tag{5.10}$$
```
> simplify(%);
```
$$24 \tag{5.11}$$
```
> x; y := 'y';  # Katsotaan x ja vapautetaan y
```
$$x$$
$$y := y \tag{5.12}$$
```
> with(plots):  # Lisägrafiikkapakkaus, tarvitaan display
> ellipsi
```
$$9\,x^2 + 16\,y^2 = 144 \tag{5.13}$$
```
> ellkuva := implicitplot(ellipsi, x = -5..5, y = -5..5, scaling = constrained);
```
$$ellkuva := PLOT(\ldots) \tag{5.14}$$
```
> ellkuva
```



> *y*   # *Tarkistetaan y:n arvo.*

$$y \tag{5.15}$$

> *y0* := *subs*(*x* = *x0*, *Y*[1]);

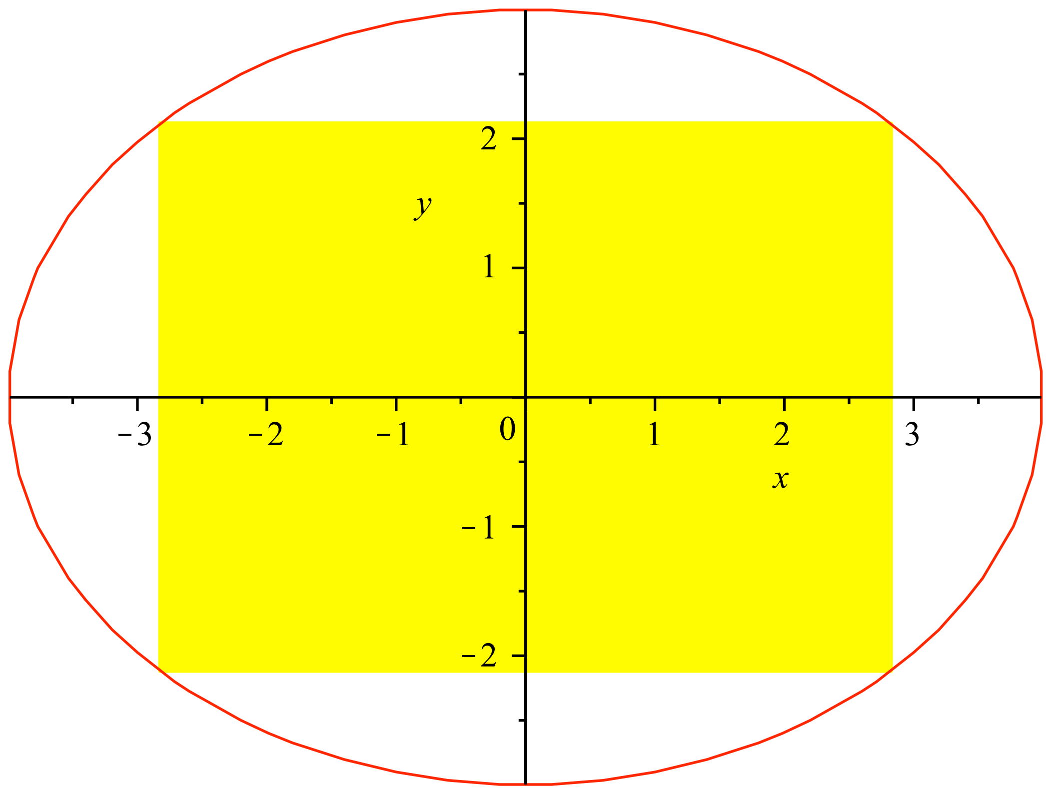
$$y0 := \frac{3}{4}\sqrt{8} \tag{5.16}$$

> *suorak* := [[*x0*, *y0*], [−*x0*, *y0*], [−*x0*, −*y0*], [*x0*, −*y0*], [*x0*, *y0*]];

$$suorak := \left[\left[2\sqrt{2}, \frac{3}{4}\sqrt{8}\right], \left[-2\sqrt{2}, \frac{3}{4}\sqrt{8}\right], \left[-2\sqrt{2}, -\frac{3}{4}\sqrt{8}\right], \left[2\sqrt{2}, -\frac{3}{4}\sqrt{8}\right], \left[2\sqrt{2}, \frac{3}{4}\sqrt{8}\right]\right] \tag{5.17}$$

> *skKuva* := *plot*(*suorak*, *filled* = *true*, *color* = *yellow*) :

> *display*(*ellkuva*, *skKuva*, *scaling* = *constrained*);
    # *scaling= .. ei tarvitse toistaa, jos(kun) annettiin yllä*

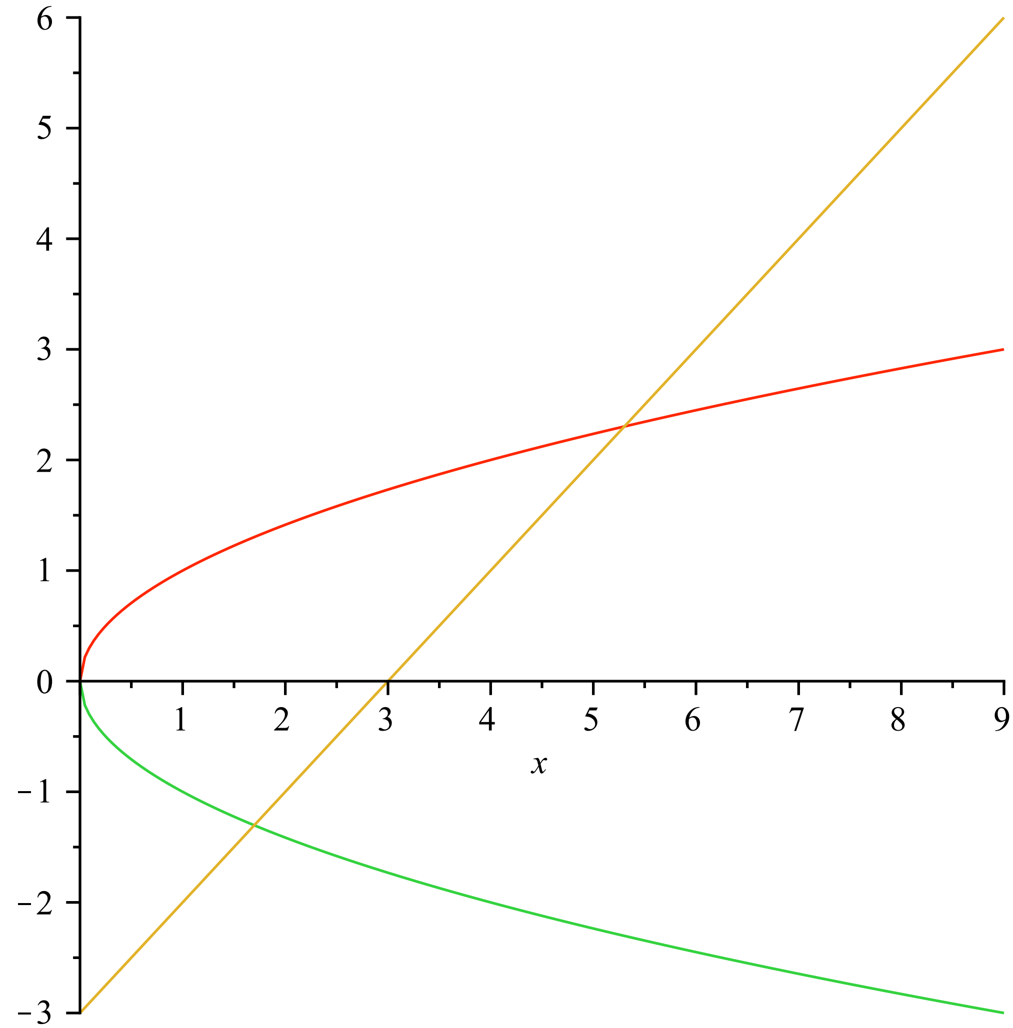


>

2.

Paraabelin $y^2 = x$ ja suoran $x - y = 3$ rajoittaman alueen pinta-ala?

> $plot\left(\left[\sqrt{x}, -\sqrt{x}, x-3\right], x = 0..9\right)$



> $paraabeli := y^2 = x$

$$paraabeli := y^2 = x \quad \textbf{(6.1)}$$

> $suora := x - y = 3$

$$suora := x - y = 3 \quad \textbf{(6.2)}$$

> $solve(\{paraabeli, suora\}, \{x, y\})$

$$\{x = RootOf(\_Z^2 - \_Z - 3) + 3, y = RootOf(\_Z^2 - \_Z - 3)\} \quad \textbf{(6.3)}$$

> $ratk := map(allvalues, \%);$

$$ratk := \left\{x = \frac{7}{2} - \frac{1}{2}\sqrt{13}, x = \frac{7}{2} + \frac{1}{2}\sqrt{13}, y = \frac{1}{2} - \frac{1}{2}\sqrt{13}, y = \frac{1}{2} + \frac{1}{2}\sqrt{13}\right\} \quad \textbf{(6.4)}$$

> $ratk1 := ratk[[1, 3]]$

$$ratk1 := \left\{x = \frac{7}{2} - \frac{1}{2}\sqrt{13}, y = \frac{1}{2} - \frac{1}{2}\sqrt{13}\right\} \quad \textbf{(6.5)}$$

> $ratk2 := ratk[[2, 4]]$

$$ratk2 := \left\{x = \frac{7}{2} + \frac{1}{2}\sqrt{13}, y = \frac{1}{2} + \frac{1}{2}\sqrt{13}\right\} \quad \textbf{(6.6)}$$

>

Valittiin sill"a perusteella, ett"a pienemm"an x:n kanssa on negat. y.
Huom! Tyoarkkia uudelleen ajettaessa ratk-joukon alkioiden j"arjestys saattaa vaihtua!

> $a := subs(ratk1, x);\ b := subs(ratk1, y);$

$$a := \frac{7}{2} - \frac{1}{2}\sqrt{13}$$

$$b := \frac{1}{2} - \frac{1}{2}\sqrt{13} \qquad (6.7)$$

> $c := subs(ratk2, x);\ d := subs(ratk2, y);$

$$c := \frac{7}{2} + \frac{1}{2}\sqrt{13}$$

$$d := \frac{1}{2} + \frac{1}{2}\sqrt{13} \qquad (6.8)$$

> $ala := \int_b^d \left((y+3) - y^2 \right) \mathrm{d}y$

$$ala := -\frac{1}{3}\left(\frac{1}{2} + \frac{1}{2}\sqrt{13}\right)^3 + \frac{1}{3}\left(\frac{1}{2} - \frac{1}{2}\sqrt{13}\right)^3 + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\sqrt{13}\right)^2 - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}\sqrt{13}\right)^2 + 3\sqrt{13} \qquad (6.9)$$

> $simplify(ala);$

$$\frac{13}{6}\sqrt{13} \qquad (6.10)$$

>

7. DokuT

> **read**("/Users/heikki/opetus/peruskurssi/v2-3/maple/v202.mpl");

> ?*interp*

> $with(plots):$

> $xd := linspace(0, 3, 7);$

$$xd := \left[0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right] \qquad (7.1)$$

> $xd := evalf(xd);$

$$xd := [0., 0.5000000000, 1., 1.500000000, 2., 2.500000000, 3.] \qquad (7.2)$$

> $f := x \rightarrow \cos(1 + x^2)$

$$f := x \rightarrow \cos(1 + x^2) \qquad (7.3)$$

> $yd := f{\sim}(xd)$

$$yd := [0.5403023059, 0.3153223624, -0.4161468365, -0.9941296761, 0.2836621855, 0.5679241733, -0.8390715291] \qquad (7.4)$$

> $p := interp(xd, yd, x);$

$$p := 1.093073361\, x^6 - 9.689758380\, x^5 + 31.38122493\, x^4 - 44.99979274\, x^3 \qquad (7.5)$$

$+ 27.62003430\, x^2 - 6.361230612\, x + 0.5403023059$

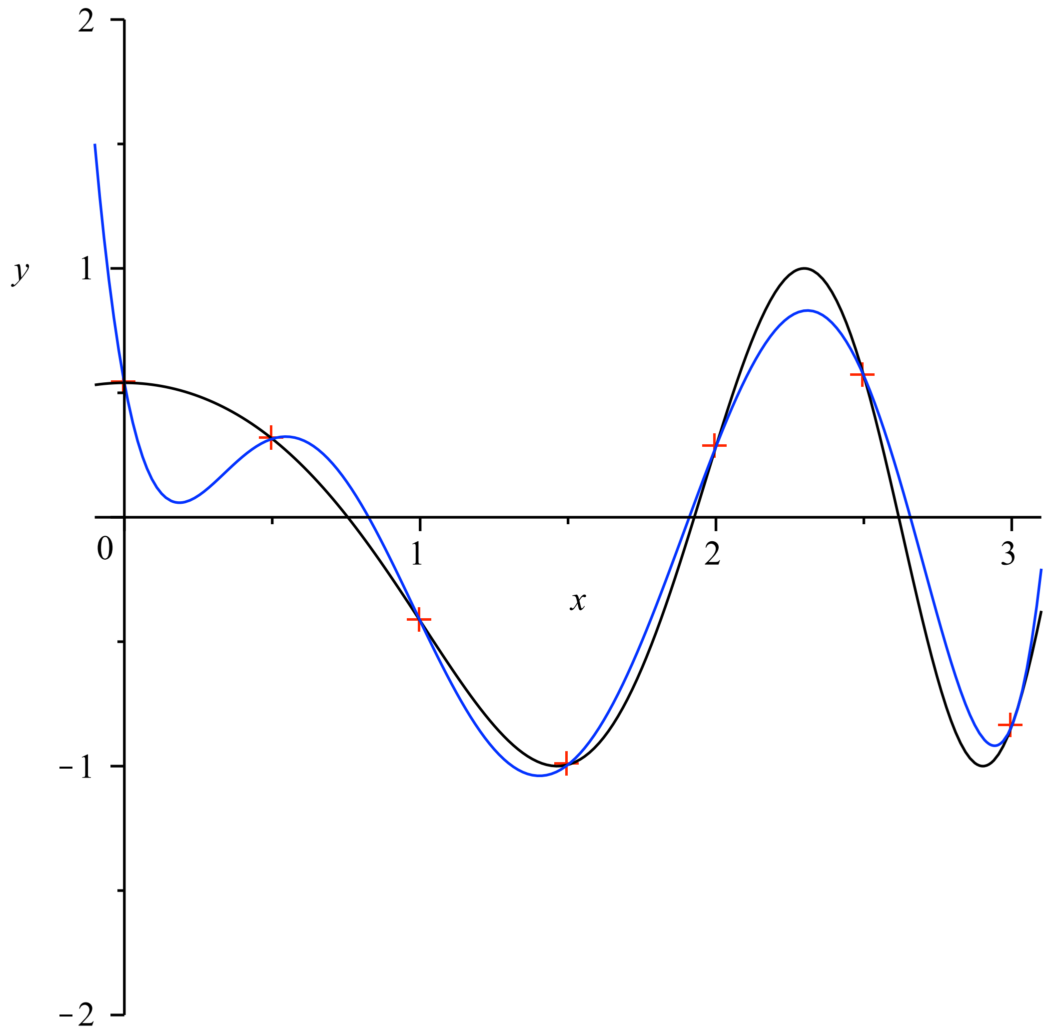
> $datakuva := plot(xd, yd, style = point, symbol = cross, symbolsize = 18)$

$$datakuva := PLOT(\ldots) \quad \textbf{(7.6)}$$

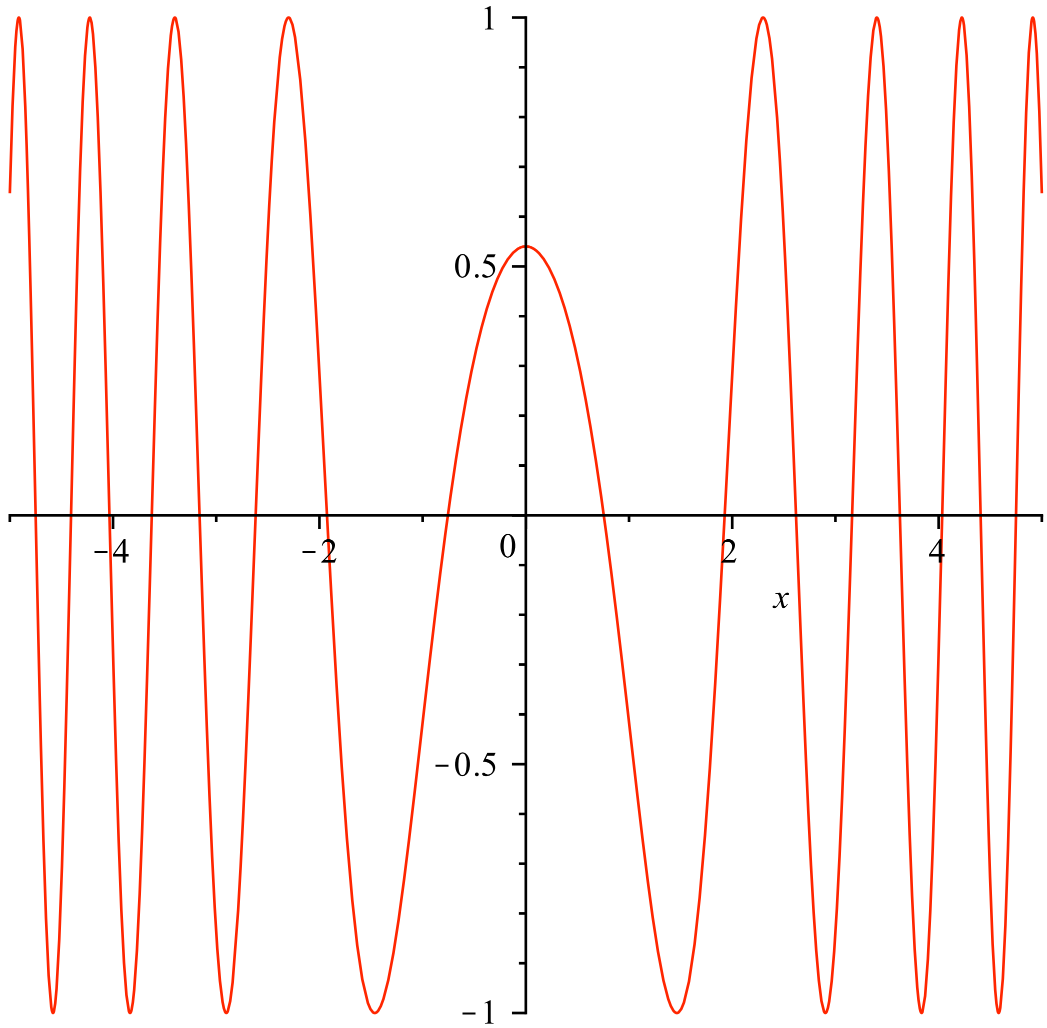
> $fkuva := plot([f(x), p(x)], x = -.1 ..3.1, y = -2 ..2, color = [black, blue])$

$$fkuva := PLOT(\ldots) \quad \textbf{(7.7)}$$

> $display(datakuva, fkuva)$



> $plot\left(\cos\left(1 + x^2\right), x = -5 ..5\right)$



> $d7f := diff(f(x), x\$7)$

$$d7f := 128\sin(1+x^2)\,x^7 - 1344\cos(1+x^2)\,x^5 - 3360\sin(1+x^2)\,x^3 + 1680\cos(1+x^2)\,x \qquad \textbf{(7.8)}$$

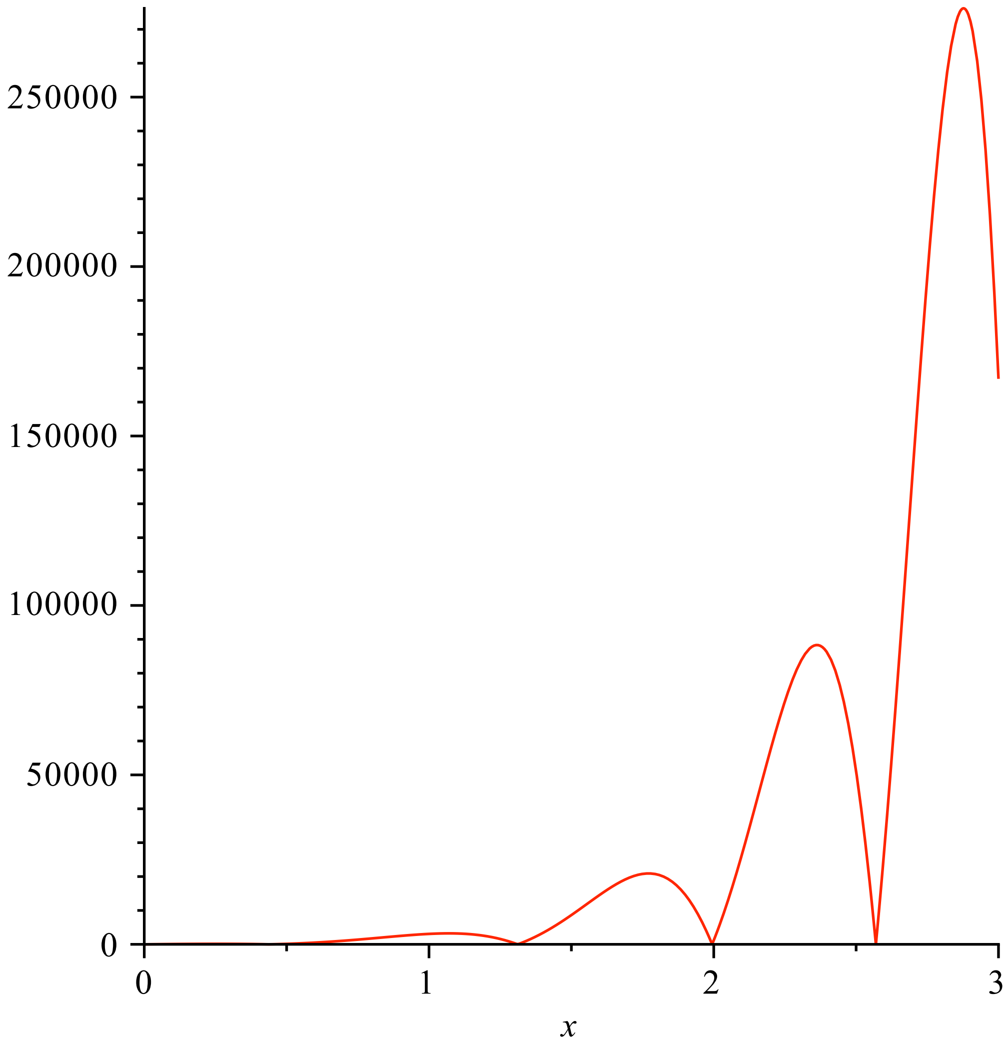
> $lprint(d7f);$

```
128*sin(1+x^2)*x^7-1344*cos(1+x^2)*x^5-3360*sin(1+x^2)*x^3+1680*
cos(1+x^2)*x
```

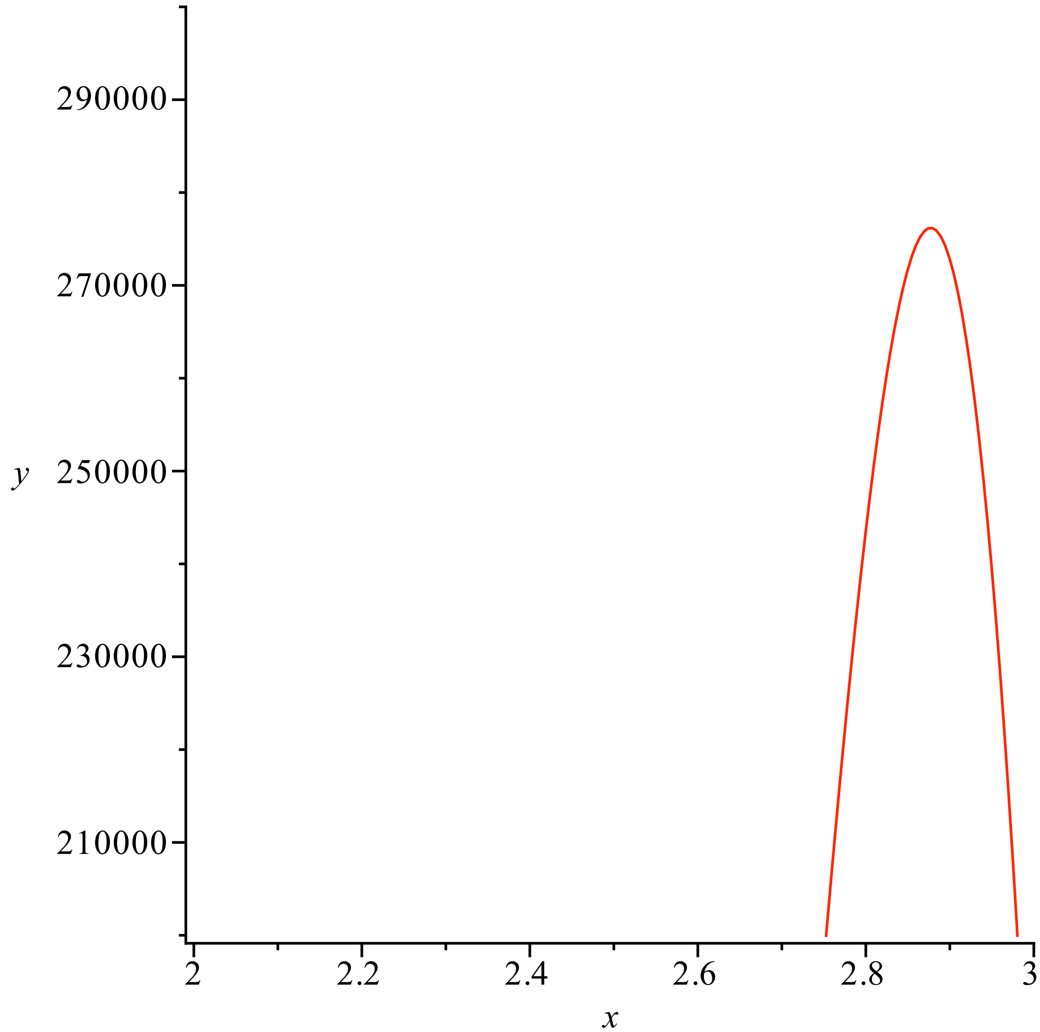
> $x\$7$

$$x, x, x, x, x, x, x \qquad \textbf{(7.9)}$$

> $plot(\mathrm{abs}(d7f), x = 0..3);$



> $plot(\mathrm{abs}(d7f), x = 2..3, y = 200000..300000);$



> maxder7 := 276000

$$maxder7 := 276000 \qquad (7.10)$$

> 7!

$$5040 \qquad (7.11)$$

> kerroin := $\frac{maxder7}{7!}$

$$kerroin := \frac{1150}{21} \qquad (7.12)$$

> evalf(%)

$$54.76190476 \qquad (7.13)$$

> tulo := product(a[j], j = 1 ..2)

$$tulo := \left(\frac{7}{2} - \frac{1}{2}\sqrt{13}\right)_1 \left(\frac{7}{2} - \frac{1}{2}\sqrt{13}\right)_2 \qquad (7.14)$$

> tulo := x→product((x − xd[j]), j = 1 ..7);

$$tulo := x \rightarrow \prod_{j=1}^{7} \left(x - xd_j\right) \qquad (7.15)$$

> $tulo(x);$

$x\,(x-0.5000000000)\,(x-1.)\,(x-1.500000000)\,(x-2.)\,(x-2.500000000)\,(x-3.)$ **(7.16)**

> $dt := diff(tulo(x), x)$

$dt := (x-0.5000000000)\,(x-1.)\,(x-1.500000000)\,(x-2.)\,(x-2.500000000)\,(x-3.) + x\,(x-1.)\,(x-1.500000000)\,(x-2.)\,(x-2.500000000)\,(x-3.) + x\,(x-0.5000000000)\,(x-1.500000000)\,(x-2.)\,(x-2.500000000)\,(x-3.) + x\,(x-0.5000000000)\,(x-1.)\,(x-2.)\,(x-2.500000000)\,(x-3.) + x\,(x-0.5000000000)\,(x-1.)\,(x-1.500000000)\,(x-2.500000000)\,(x-3.) + x\,(x-0.5000000000)\,(x-1.)\,(x-1.500000000)\,(x-2.)\,(x-3.) + x\,(x-0.5000000000)\,(x-1.)\,(x-1.500000000)\,(x-2.)\,(x-2.500000000)$ **(7.17)**

> $nollak := solve(dt = 0);$

$nollak := 0.1609812071, 2.839018793, 0.7021089813, 2.297891019, 1.234672665, 1.765327335$ **(7.18)**

> $tulo\sim([nollak])$

[0.7487648688, -0.7487648684, -0.1808515128, 0.1808515126, 0.09655295638, -0.09655295646] **(7.19)**

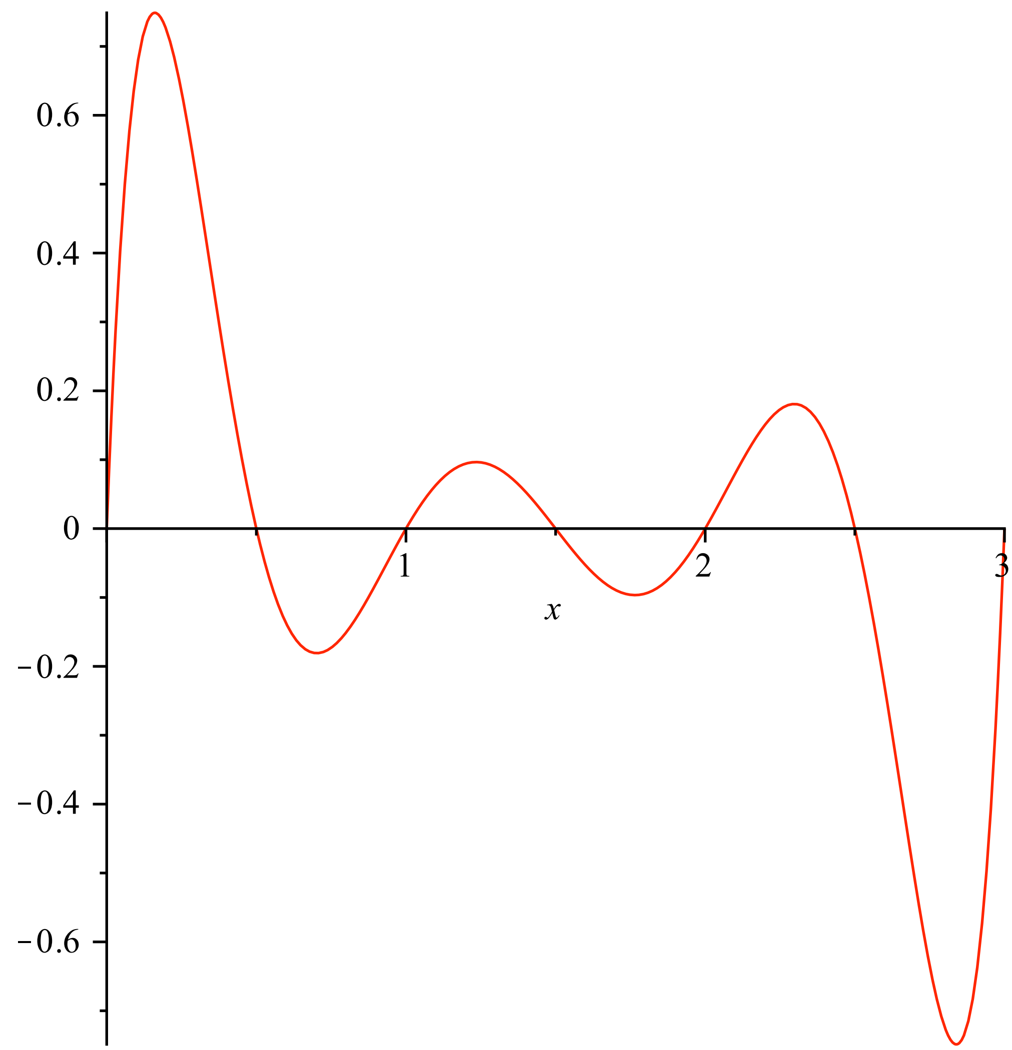
> abs~(%)

[0.7487648688, 0.7487648684, 0.1808515128, 0.1808515126, 0.09655295638, 0.09655295646] **(7.20)**

> $maxtulo :=$ max(%)

$maxtulo := 0.7487648688$ **(7.21)**

> $plot(tulo(x), x = 0..3);$



> $M := kerroin \cdot maxtulo$

$$M := 41.00379043 \quad \textbf{(7.22)}$$

> $virhe := f(x) - p$

$$virhe := \cos(1 + x^2) - 1.093073361\, x^6 + 9.689758380\, x^5 - 31.38122493\, x^4 + 44.99979274\, x^3 - 27.62003430\, x^2 + 6.361230612\, x - 0.5403023059 \quad \textbf{(7.23)}$$

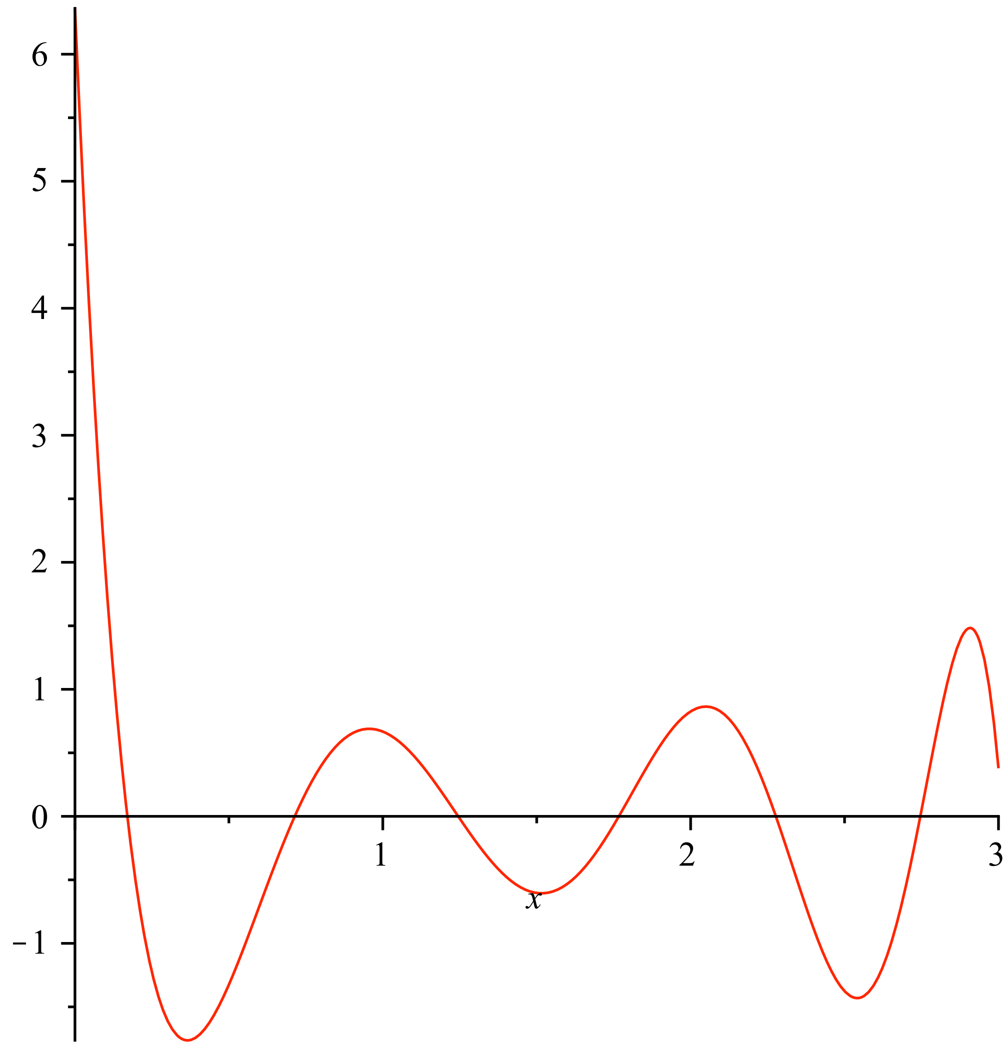
> $dvirhe := diff(virhe, x)$

$$dvirhe := -2 \sin(1 + x^2)\, x - 6.558440166\, x^5 + 48.44879190\, x^4 - 125.5248997\, x^3 + 134.9993782\, x^2 - 55.24006860\, x + 6.361230612 \quad \textbf{(7.24)}$$

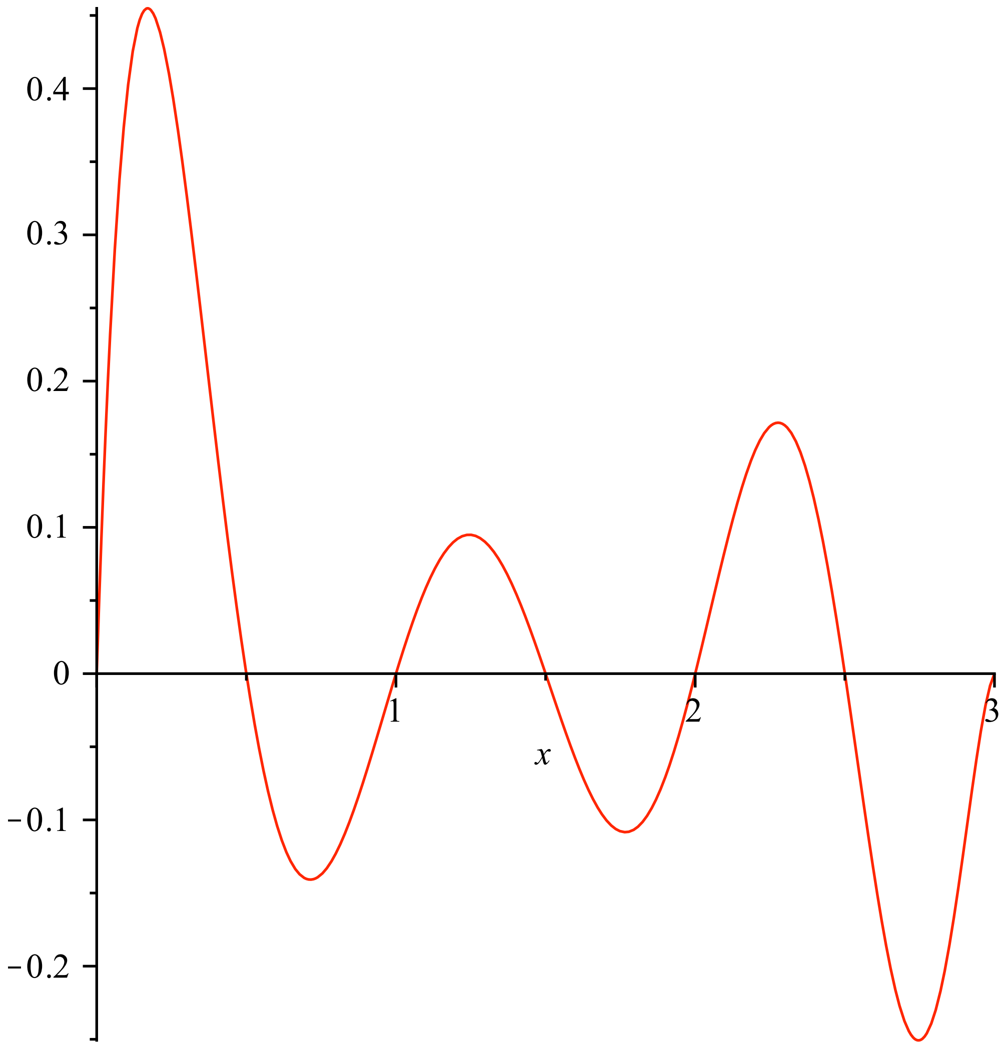
> $maxx := fsolve(dvirhe = 0, x = 0.2);$

$$maxx := 0.1701634433 \quad \textbf{(7.25)}$$

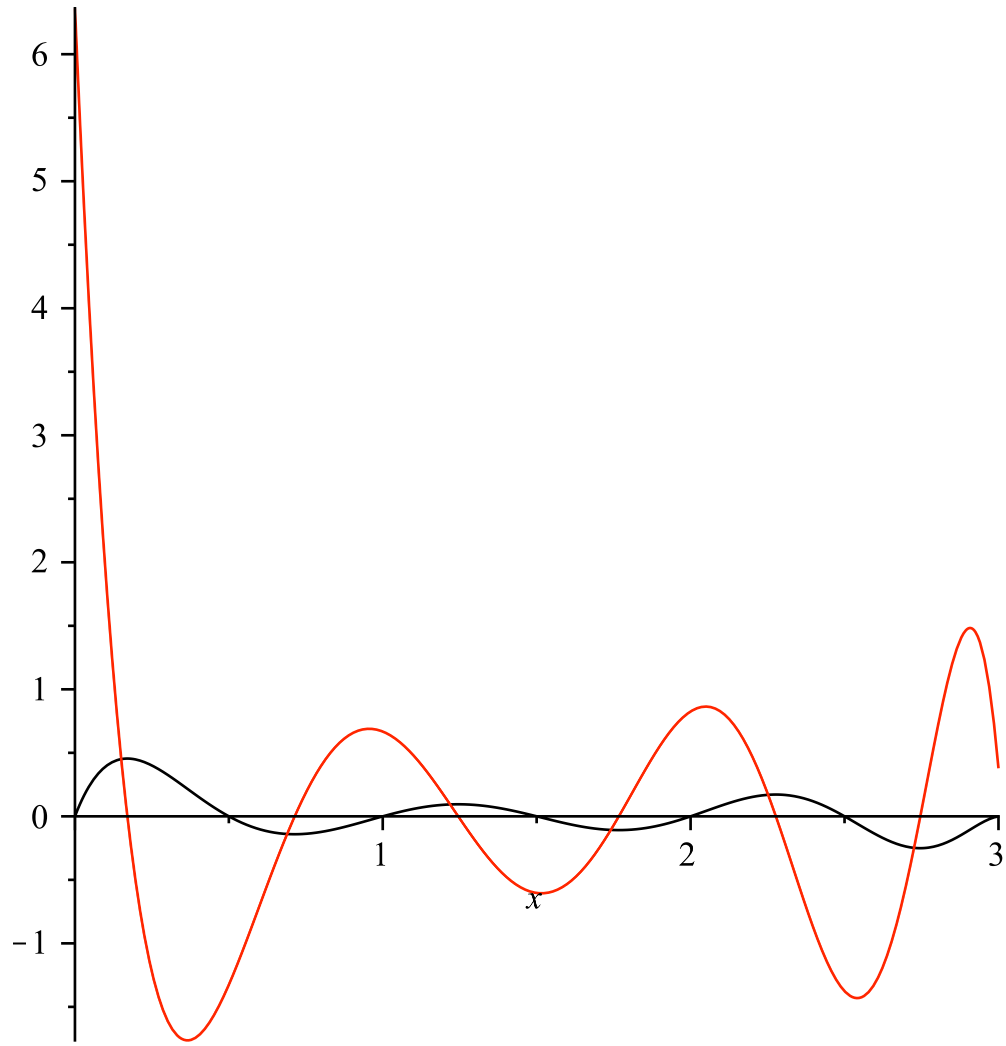
> $plot(dvirhe, x = 0 .. 3)$



> $plot(virhe, x = 0..3)$



> $plot([virhe, dvirhe], x = 0..3, color = [black, red])$



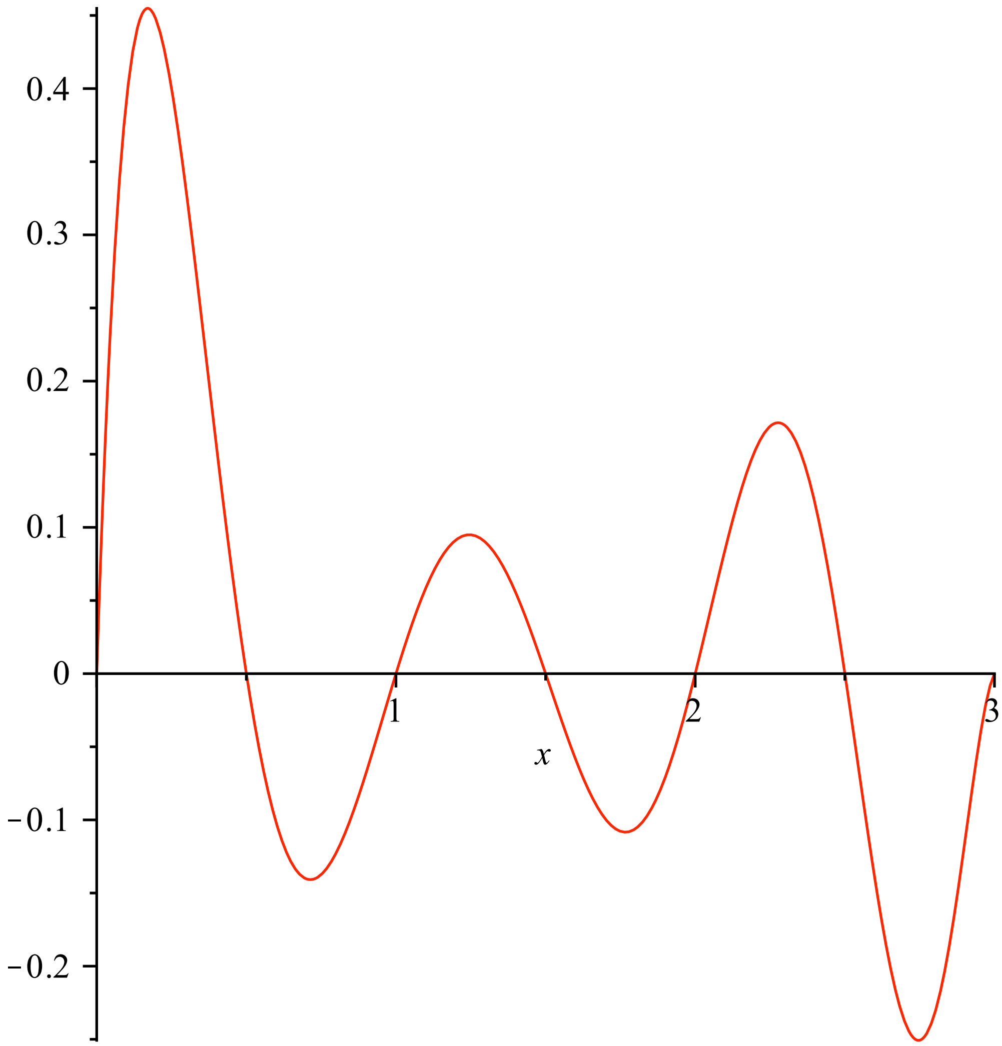
> *subs*($x = maxx, virhe$);
Tmaxv := *evalf*(%)   # *Todellinen max-virhe.*

$$\cos(1.028955597) - 0.0608406869$$

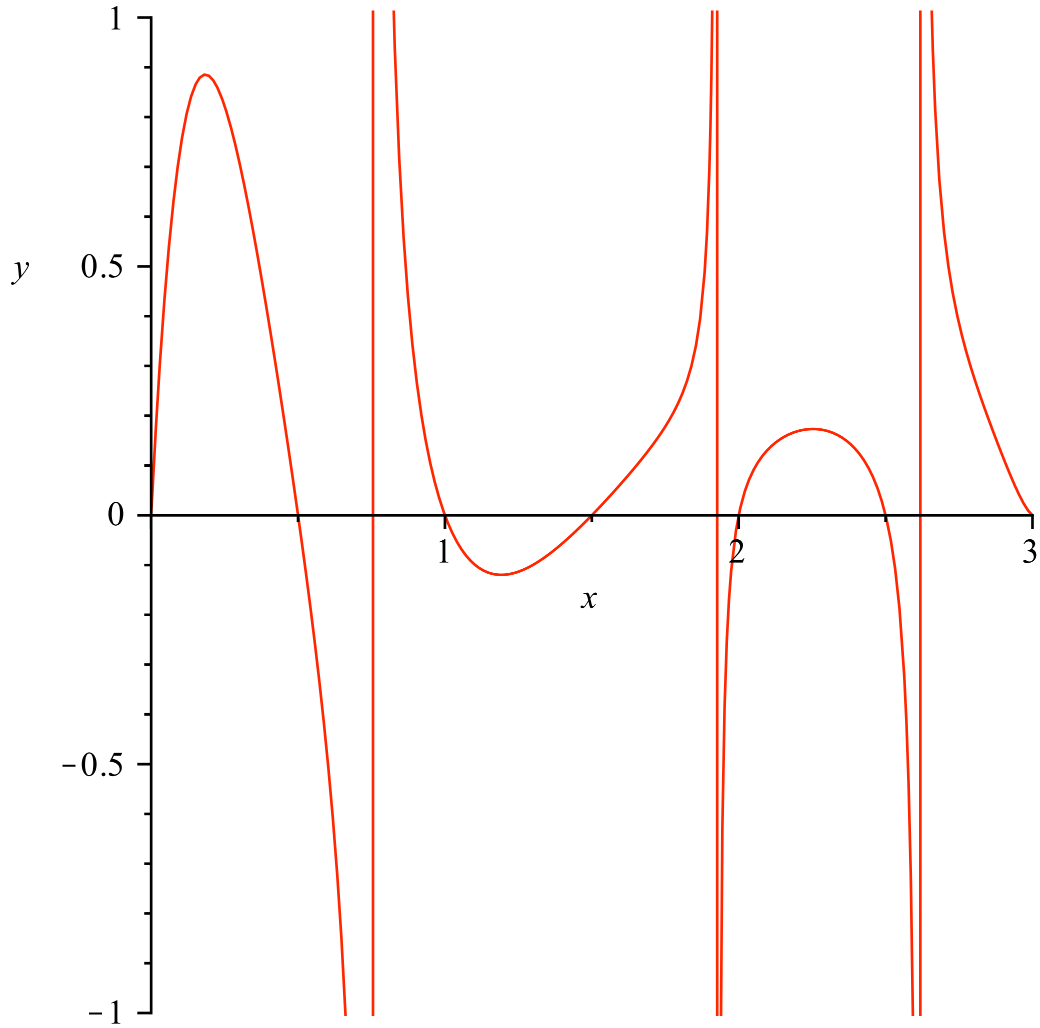
$$Tmaxv := 0.4548732428 \qquad \textbf{(7.26)}$$

Virhearvio max-virheen suhteen (ja muutenkin) on t"ass"a tapauksessa k"aytt"okelvottoman karkea, johtuen 7. derivaatan valtavasta maksimista. (Eih"an se ξ v"altt"am"att"a (l"ahimainkaan) siihen max-kohtaan osu, mutta kun siit"a ei mit"a"an tiedet"a, ei yleisell"a kaavalla parempaa arviota max-virheelle saada.)

> *plot*($f(x) - p, x = 0..3$)



> $plot\left(\frac{f(x) - p}{f(x)}, x = 0..3, y = -1..1\right)$



>

8.

> $N := x \rightarrow evalf\left(x - \frac{f(x)}{\mathrm{D}(f)(x)}\right)$

$$N := x \rightarrow evalf\left(x - \frac{f(x)}{\mathrm{D}(f)(x)}\right) \quad \textbf{(8.1)}$$

>

> $f := x \rightarrow x \cdot \cos(x) - \sin(x) - 1;$

$$f := x \rightarrow x \cos(x) - \sin(x) - 1 \quad \textbf{(8.2)}$$

> $fkuva := plot(f(x), x = 0..3 \cdot \mathrm{Pi}, color = black);$

$$fkuva := PLOT(\ldots) \quad \textbf{(8.3)}$$

> $N(x);$

$$x + \frac{x \cos(x) - 1.\sin(x) - 1.}{x \sin(x)} \quad \textbf{(8.4)}$$

> $x[0] := \mathrm{Pi} + .3;$

(8.5)

$$x_0 := \pi + 0.3 \quad (8.5)$$

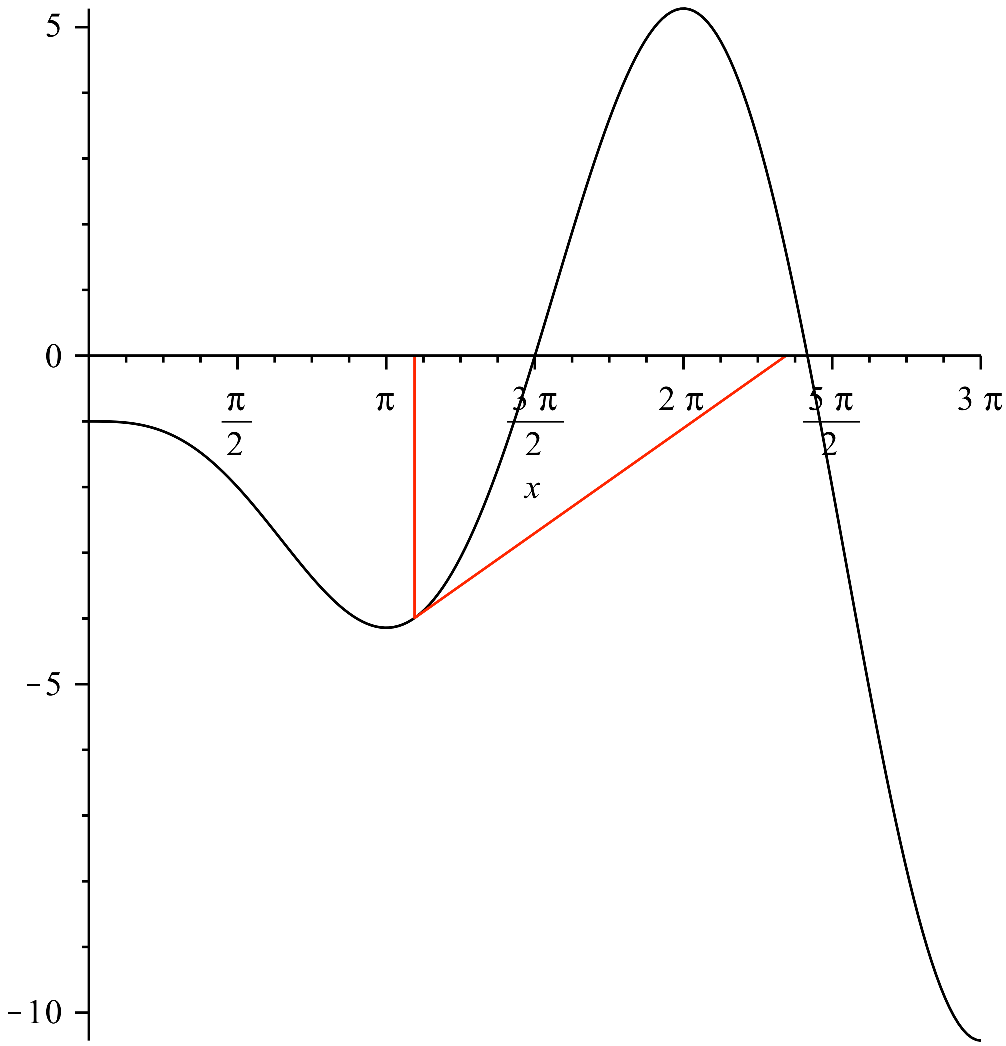
```
> for k from 1 to 10 do
   x[k] := N(x[k - 1])
  end do
```

$$x_1 := 7.366983641$$
$$x_2 := 7.607183306$$
$$x_3 := 7.592100585$$
$$x_4 := 7.592056182$$
$$x_5 := 7.592056182$$
$$x_6 := 7.592056182$$
$$x_7 := 7.592056182$$
$$x_8 := 7.592056182$$
$$x_9 := 7.592056182$$
$$x_{10} := 7.592056182 \quad (8.6)$$

```
> tangkuva := x0→plot([[x0, 0], [x0, f(x0)], [N(x0), 0]])
```

$$tangkuva := x0 \rightarrow plot([[x0, 0], [x0, f(x0)], [N(x0), 0]]) \quad (8.7)$$

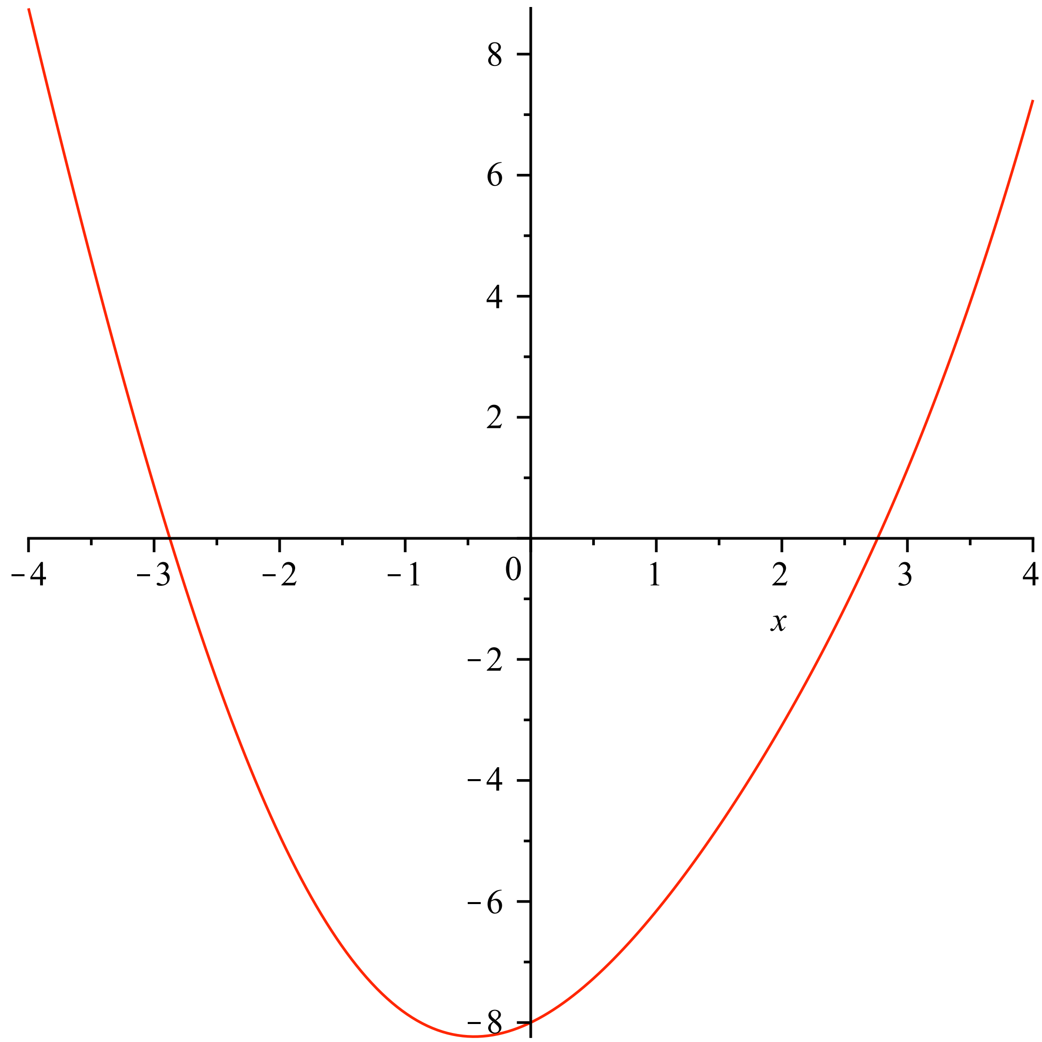
```
> #tang:= (f, x0, x)→f(x0) + D(f)(x0)·(x − x0)
>
> display(seq(display(fkuva, tangkuva(x[k − 1])), k = 1 ..9), insequence = true)
```



> $f := x \rightarrow x^2 + \sin(x) - 8$

$$f := x \rightarrow x^2 + \sin(x) - 8 \quad (8.8)$$

> $plot(f(x), x = -4 .. 4)$



> $x[0] := -1$

$$x_0 := -1 \quad \textbf{(8.9)}$$

```
> for k from 1 to 10 do
  x[k] := N(x[k − 1])
  end do
```

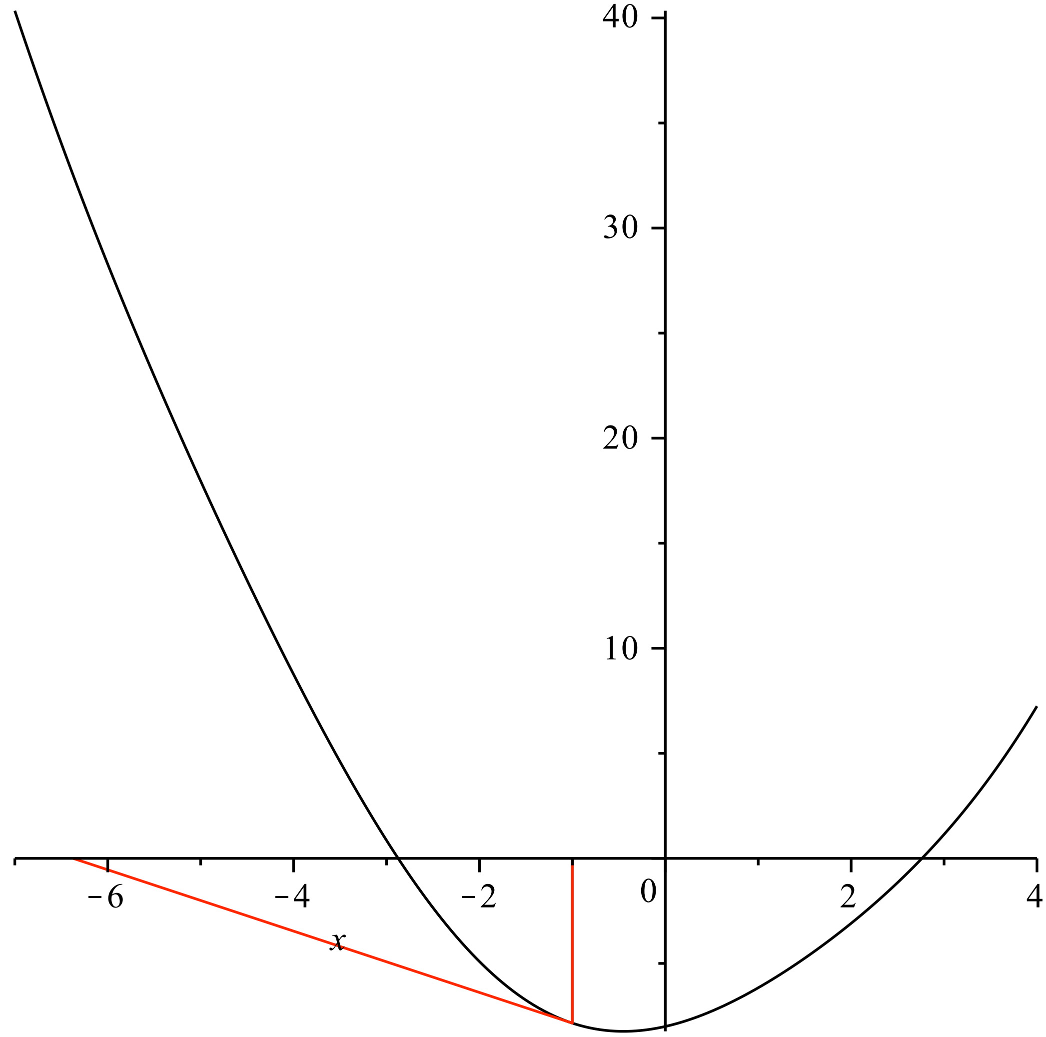
$$x_1 := -6.371982855$$
$$x_2 := -3.604384472$$
$$x_3 := -2.933318548$$
$$x_4 := -2.875234609$$
$$x_5 := -2.874675521$$
$$x_6 := -2.874675468$$
$$x_7 := -2.874675468$$
$$x_8 := -2.874675468$$
$$x_9 := -2.874675468$$

$$x_{10} := -2.874675468 \quad (8.10)$$

> $fkuva := plot(f(x), x = -7..4, color = black);$

$$fkuva := PLOT(...) \quad (8.11)$$

> $display(seq(display(fkuva, tangkuva(x[k-1])), k = 1..9), insequence = true)$



>

10. Diffyht.

> $restart:$

> $diffyhtalo := \frac{d}{dx} y(x) - y(x) = \cos(x)$

$$diffyhtalo := \frac{d}{dx} y(x) - y(x) = \cos(x) \quad (10.1)$$

> $AE := y(0) = 1$

$$AE := y(0) = 1 \quad (10.2)$$

> $ratk := dsolve(\{diffyhtalo, y(0) = 1\}, y(x));$

$$ratk := y(x) = -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + \frac{3}{2}e^{x} \quad \textbf{(10.3)}$$

> $Y := subs(ratk, y(x));$

$$Y := -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + \frac{3}{2}e^{x} \quad \textbf{(10.4)}$$

> $subs(y(x) = Y, diffyhtalo);$

$$\frac{d}{dx}\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + \frac{3}{2}e^{x}\right) + \frac{1}{2}\cos(x) - \frac{1}{2}\sin(x) - \frac{3}{2}e^{x} = \cos(x) \quad \textbf{(10.5)}$$

> $eval(\%);$

$$\cos(x) = \cos(x) \quad \textbf{(10.6)}$$

> $eval(Y, x = 0);$

$$1 \quad \textbf{(10.7)}$$

b)

> $ratk := dsolve(\{diffyhtalo, y(0) = c\}, y(x));$

$$ratk := y(x) = -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + e^{x}\left(c + \frac{1}{2}\right) \quad \textbf{(10.8)}$$

> $Y := rhs(ratk)$

$$Y := -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + e^{x}\left(c + \frac{1}{2}\right) \quad \textbf{(10.9)}$$

> $C := [seq(-1 + 0.1 \cdot k, k = 1 ..10)]$

$$C := [-0.9, -0.8, -0.7, -0.6, -0.5, -0.4, -0.3, -0.2, -0.1, 0.] \quad \textbf{(10.10)}$$

> $Yparvi := seq(Y, c = C)$

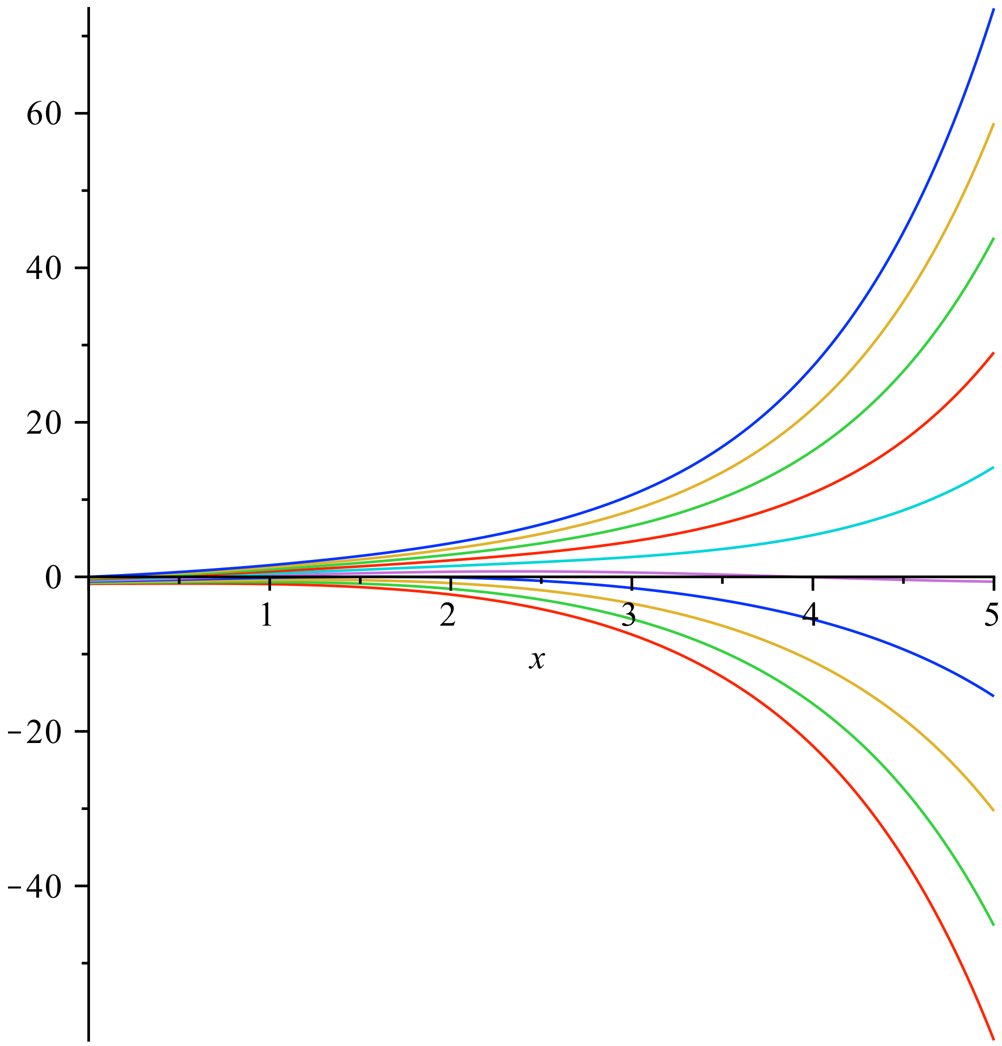
$$Yparvi := -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) - 0.4000000000\,e^{x}, -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) - 0.3000000000\,e^{x}, -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) - 0.2000000000\,e^{x}, -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) - 0.1000000000\,e^{x}, -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x), -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + 0.1000000000\,e^{x}, -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + 0.2000000000\,e^{x}, -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + 0.3000000000\,e^{x}, -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + 0.4000000000\,e^{x}, -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + 0.5000000000\,e^{x} \quad \textbf{(10.11)}$$

> varit := "AliceBlue", "Aqua", "Azure", "Black", "Brown"

varit := "AliceBlue", "Aqua", "Azure", "Black", "Brown" **(10.12)**

> #plot([Yparvi], x = 0 ..5, color = [varit, varit])

> plot([Yparvi], x = 0 ..5)



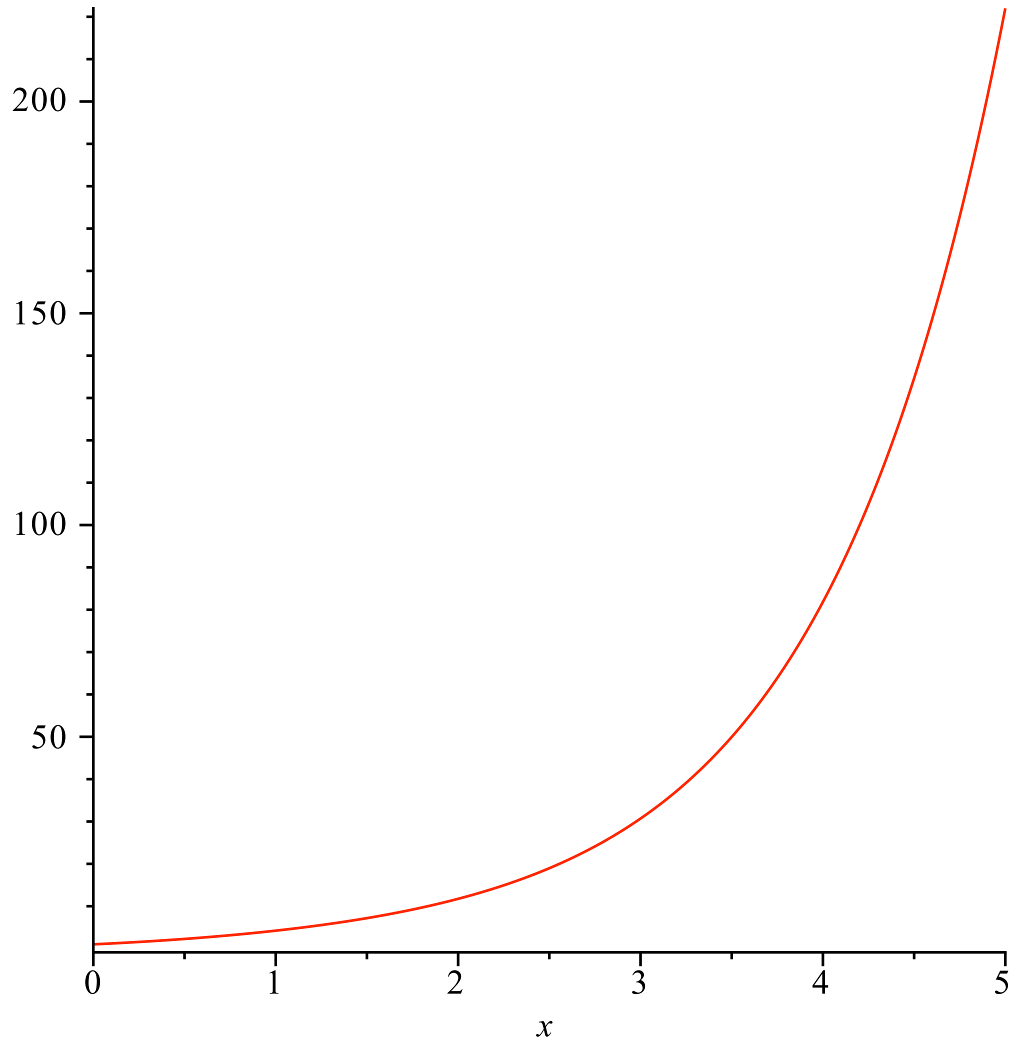
>

> $with(plots):$

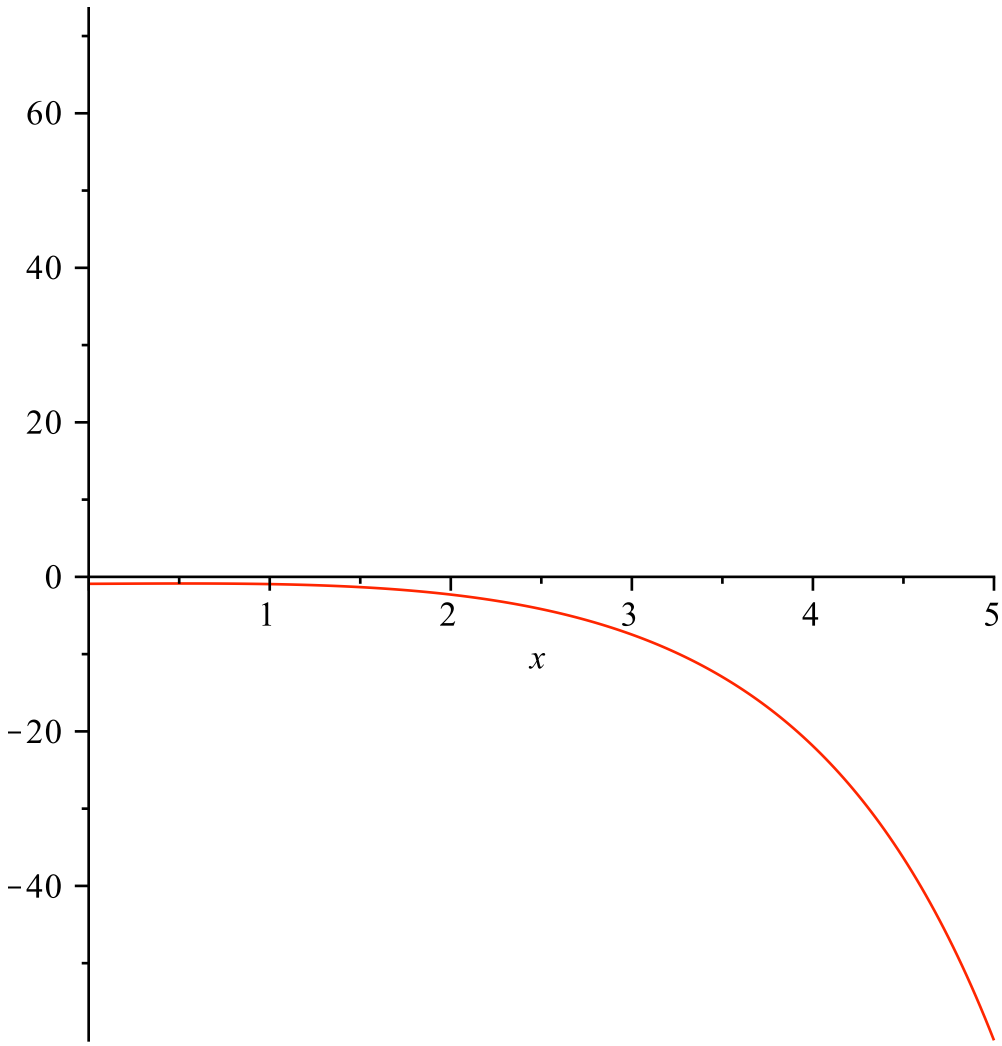
> $kayra := (K, a, b) \rightarrow plot(subs(c = K, Y), x = a..b)$

$$kayra := (K, a, b) \rightarrow plot(subs(c = K, Y), x = a..b) \tag{10.13}$$

> $display(kayra(1, 0, 5))$



> $display(seq(kayra(K, 0, 5), K = C), insequence = true)$



14. DokuT

```
> restart :
> with(plots) :
> #read("/Users/heikki/opetus/peruskurssi/v2-3/maple/v202.mpl");
> linspace := (a, b, n) → [seq(a + iii * (b − a) / (n − 1), iii = 0 ..n - 1)]
```

$$linspace := (a, b, n) \rightarrow \left[seq\left(a + \frac{iii\ (b-a)}{n-1}, iii = 0\,..n-1\right)\right] \quad \textbf{(11.1)}$$

```
>
> td := linspace(1900, 1990, 10)
```

$$td := [1900, 1910, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990] \quad \textbf{(11.2)}$$

```
> yd := [76, 92, 106, 122, 132, 150, 179, 203, 226, 248]
```

$$yd := [76, 92, 106, 122, 132, 150, 179, 203, 226, 248] \quad \textbf{(11.3)}$$

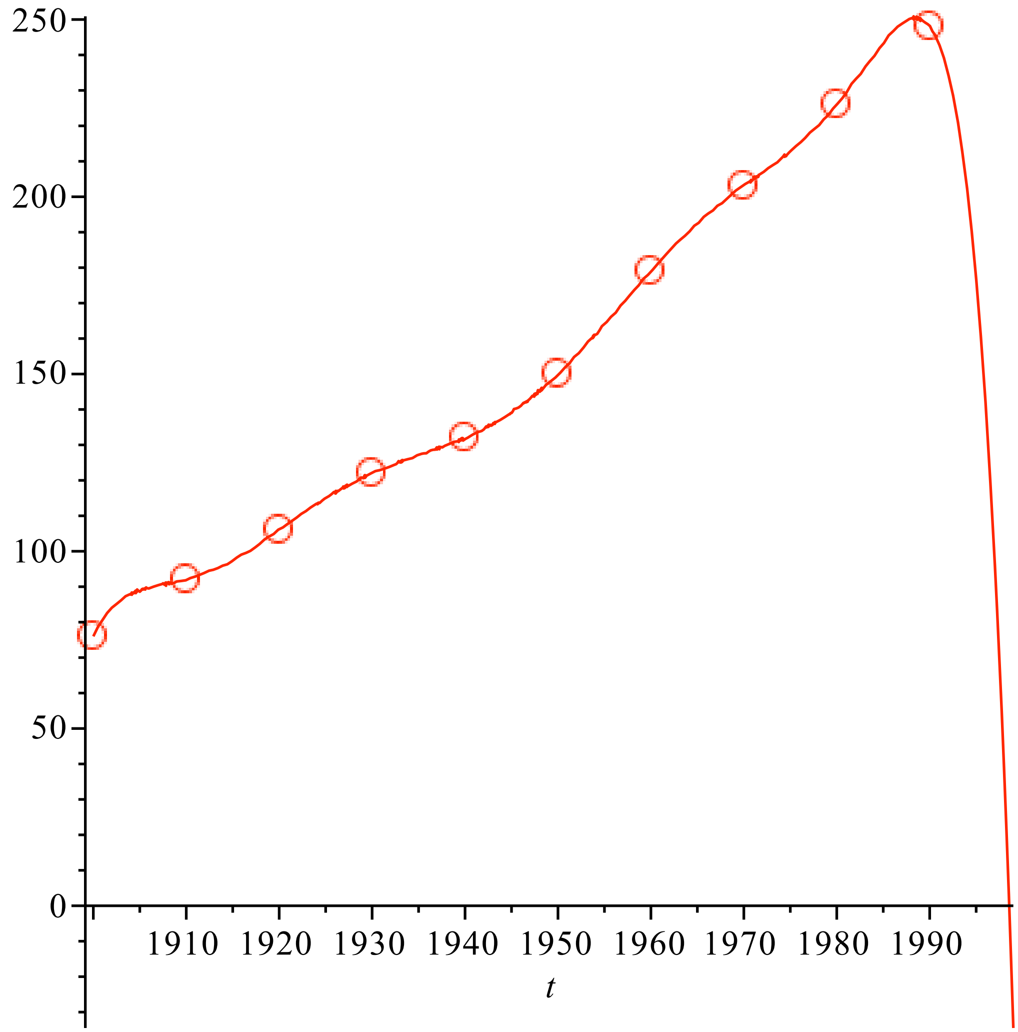
```
> Digits := 20
```

$$Digits := 20 \quad \textbf{(11.4)}$$

> $p := interp(td, yd, t)$

$$p := -\frac{31}{181440000000000} t^9 + \frac{1993}{672000000000} t^8 - \frac{6918319}{302400000000} t^7 + \frac{3293971}{32000000} t^6 - \frac{257218344031}{864000000} t^5 + \frac{5509840052131}{9600000} t^4 - \frac{33456538331517239}{45360000} t^3 + \frac{20471669634936287}{33600} t^2 - \frac{184938855034179055\,1}{6300} t + 62853427235011914 \quad \textbf{(11.5)}$$

> $display(plot(p, t = 1900..1999), plot(td, yd, style = point, symbol = circle, symbolsize = 20))$



Tarkkuudella Digits:10 menee aivan pipariksi.

> $with(CurveFitting):$

> $Digits;$

$$20 \quad \textbf{(11.6)}$$

> $Digits := 10$

$$Digits := 10 \quad \textbf{(11.7)}$$

> $LP := PolynomialInterpolation(td, yd, t, form = Lagrange):$

> $polykuva := plot(LP, t = 1900..1990)$

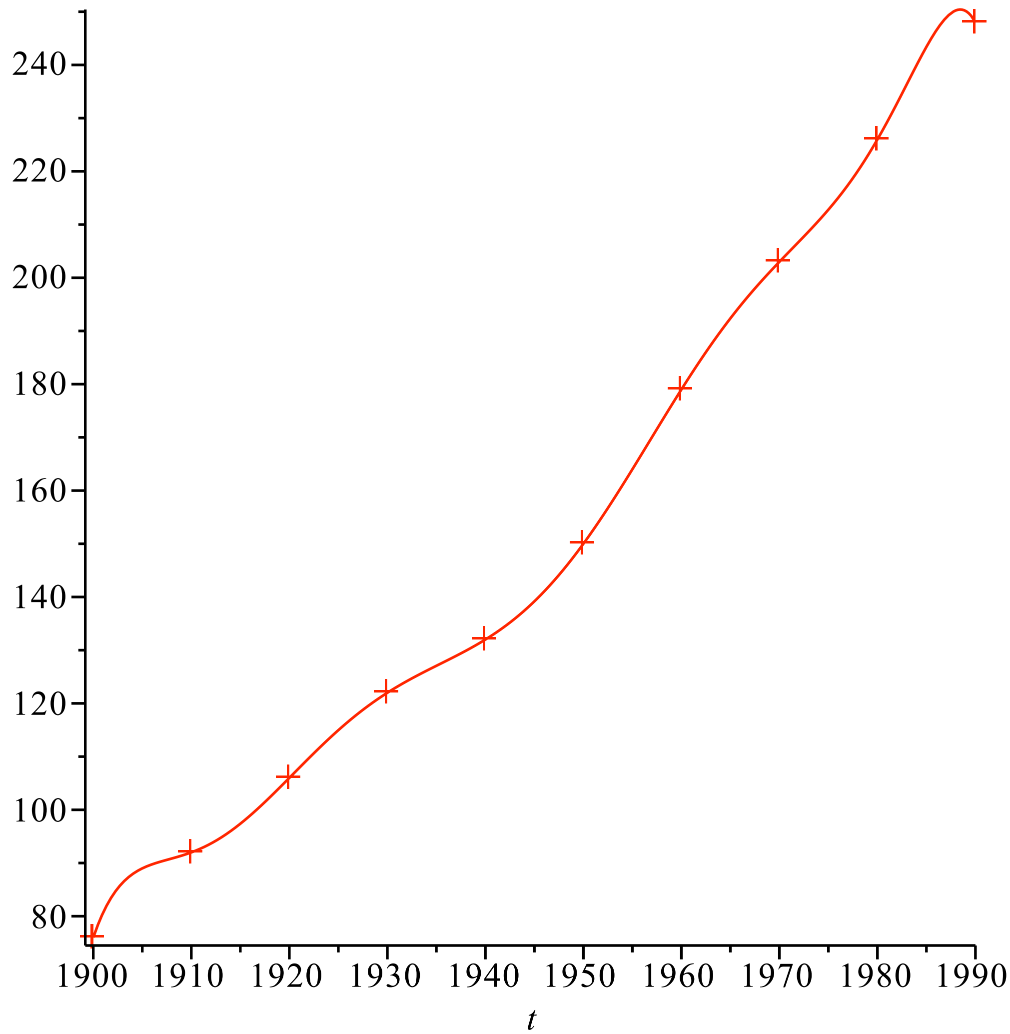
$$polykuva := PLOT(...) \quad \textbf{(11.8)}$$

>

> datakuva := plot(td, yd, style = point, symbol = cross, symbolsize = 16)

$$datakuva := PLOT(\ldots) \quad (11.9)$$

> display(polykuva, datakuva)



>

T"ass"a riitti Maplen perustarkkuus, kun k"aytettiin Lagrangen muotoa.