

Timo Salin: *The quenching problem for the N -dimensional ball*; Helsinki University of Technology Institute of Mathematics Research Reports A459 (2003).

Abstract: *In this paper we consider the quenching problem for the reaction diffusion equation $u_t - \Delta u = f(u)$ with Cauchy-Dirichlet data, in the case where the reaction term is singular at $u = 0$ in the sense that $\lim_{u \downarrow 0} f(u) = -\infty$. For $u > 0$ we take $f(u)$ to be smooth and to satisfy $(-1)^k f^{(k)}(u) < 0$, $k = 0, 1, 2$. Furthermore we assume that the reaction term is weakly singular at the origin in the sense that: $|u^n f^{(n)}(u)| = o(|f(u)|)$, as $u \downarrow 0$, $n = 1$ and $n = 2$.*

We study the equation in $\Omega = \{x \in \mathcal{R}^N : |x| < R\}$ and in the case where the initial function is radial, i.e., $u_0(x) = u_0(r)$ and $u_0'(r) \geq 0$ ($r = |x|$). We show that quenching occurs for sufficiently large Ω and prove the quenching rate estimate for the problem. We also study a refined asymptotic analysis of the solution near a quenching point for two examples of reaction terms. More precisely, we give an asymptotic expression for the solution with respect to space- and time variable in a backward space-time parabola near a quenching point.

AMS subject classifications: 35K20,35K55,35K57

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tsalin@cc.hut.fi

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Institute of Mathematics, Helsinki Univ. of Tech., Espoo, 2003

Helsinki University of Technology

Department of Engineering Physics and Mathematics

Institute of Mathematics

P.O. Box 1100, 02015 HUT, Finland

email:math@hut.fi <http://www.math.hut.fi/>