2.1 *Same birthday.* The are 23 student in a class. Let X_i be a number which tells on which day of the year the student is born, when the days are numbered from 1 to 365. Let us assume that the $X_1, \dots, X_{23}$ are independent uniformly distributed random variables in $\{1, 2, \dots, 365\}$. What is the probability that some of the students in the class share the same birthday?

2.2 *Sum and product of random bits.* Let $\theta_1, \dots, \theta_n$ be independent Bernoulli distributed random variables with parameter $p \in (0, 1)$, so that $\mathbb{P}(\theta_i = 1) = p$ and $\mathbb{P}(\theta_i = 0) = 1 - p$ for all i. Find out the distributions of the following random variablest:

(a) $X = \theta_1 + \theta_2$,
(b) $Y = \theta_1 \theta_2$,
(c) $Z = \theta_1 + \cdots + \theta_n$,
(d) $W = \theta_1 \cdots \theta_n$.

2.3 *Max ja min of random bits.* Let B_1 ja B_2 be independent uniformly distributed random variables in $\{0, 1\}$. Define $X = \min\{B_1, B_2\}$ and $Y = \max\{B_1, B_2\}$. Are X and Y dependent or independent? Explain your answer carefully.

2.4 *Conditional probabilities.* A symmetric die is thrown twise and the outcomes are denoted by X_1 and X_2. Then X_1 and X_2 are independent uniformly distributed random integers in $\{1, 2, \dots, 6\}$. Write down examples of events A ja B in terms of X_1 and X_2, where

(a) $\mathbb{P}(A \,|\, B) < \mathbb{P}(A)$,
(b) $\mathbb{P}(A \,|\, B) = \mathbb{P}(A)$,
(c) $\mathbb{P}(A \,|\, B) > \mathbb{P}(A)$.

2.5 *Independent triplets and pairs.* Let X_1, X_2, X_3 be random integers in a discrete probability space (Ω, P) Are the following statements true or false? Prove them true or show them false by giving a counterexample.

(a) If the random variables X_1, X_2, X_3 are mutually independent, then also the random variables X_i, X_j are mutually independent for all $i \neq j$.
(b) If X_i, X_j are mutually independent for all $i \neq j$, then also X_1, X_2, X_3 are mutually independent.