

You should return your solutions as a hard copy or scanned pdf to the instructor of your exercise group by **Tue 19 Nov 2013 noon**.

3.1 *Expected values of transforms.* Let X and Y be independent uniformly distributed random numbers in $\{1, 2, 3\}$. Compute the expectations of the following random numbers:

- (a) $X + Y$
- (b) $X - Y$
- (c) XY
- (d) X/Y
- (e) X^Y
- (f) $\sin(\pi/X) \cos(\pi/Y)$

3.2 *Expectation of a product.* Give examples of random numbers such that

- (a) $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y)$,
- (b) $\mathbb{E}(XY) \neq \mathbb{E}(X)\mathbb{E}(Y)$.

3.3 *Simulation of discrete distributions using random bits.* Let $\theta_1, \theta_2, \dots$ be an independent random bit sequence where θ_i is uniformly distributed in $\{0, 1\}$. Let $\phi : \{0, 1\}^4 \rightarrow \mathbb{R}$ be some function and define $X_1 = \phi(\theta_1, \dots, \theta_4)$, $X_2 = \phi(\theta_5, \dots, \theta_8)$, $X_3 = \phi(\theta_9, \dots, \theta_{12})$, etc.

- (a) Prove that the random numbers X_1, X_2, \dots are independent.
- (b) Give an example of a function ϕ such that X_1, X_2, \dots follow the uniform distribution on $\{1, \dots, 16\}$.
- (c) Give an example of a function ϕ such that X_1, X_2, \dots follow the distribution μ on $\{0, 1\}$ where $\mu(1) = 1/16$.
- (d) Does there exist a function ϕ such that X_1, X_2, \dots follow the uniform distribution on $\{1, 2, 3\}$?

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3.4 *Independent geometric random numbers.* The *geometric distribution* on $\{1, 2, \dots\}$ with success probability p is a probability function

$$\mu(k) = (1 - p)^{k-1}p, \quad k = 1, 2, \dots$$

Let X and Y be independent random numbers following a geometric distribution with success probabilities p and q , respectively. Denote $Z = \min(X, Y)$.

- (a) Prove that $\mathbb{P}(X \geq k) = (1 - p)^{k-1}$ and $\mathbb{P}(Y \geq k) = (1 - q)^{k-1}$ for all $k \geq 1$.
- (b) Prove with the help of part a) that $\mathbb{P}(Z \geq k) = (1 - r)^{k-1}$ for all $k \geq 1$ and find out a number r such that this holds.
- (c) Prove with the help of part b) that the random number Z follows the geometric distribution with success probability r .

3.5 *Monkey as a writer.* A monkey sits in front of a laptop and randomly hits a keyboard of 50 characters (caps lock removed).

- (a) Assume that during a year the monkey produces a text with a hundred million characters. How many times the word "kivi" appears in the text?
- (b) The novel *Seven Brothers* has 635 864 characters. How long text must the monkey write, so that a precise copy (ignoring capital letters) of the novel will appear in the text at least once on average?