

6.1 *Extinction of a branching process.* Consider a branching process where each individual, independently of the others, produces zero children with probability $1/8$, one child with probability $1/2$, and two children with probability $3/8$.

- (a) Compute the probability generating function $G_X(t)$ of the number of children.
- (b) Find the solutions of $G_X(t) = t$.
- (c) Compute the probability that the population becomes extinct.

6.2 *Kindergarten.* A kindergarten has 10 boys and 10 girls. Assume that each pair of boys becomes friends with probability $1/2$, each pair of girls with probability $1/3$, and each girl–boy pair with probability $1/20$.

- (a) Compute the expected number and the probability generating function of a chosen girl.
- (b) Compute the expected number and the probability generating function of a chosen boy.
- (c) Compute the probability that a chosen girl has at least 2 friends.
- (d) Compute the probability that a chosen triplet of three boys form a clique, that is, each of the three is a friend of everyone else.

6.3 *Independently connected random graph.* Consider a random graph with n nodes $V = \{1, \dots, n\}$ where each pair of nodes is connected with probability p , independently of the other nodes pairs. Denote the neighbors of node x by

$$N(x) = \{y \in V(G) : xy \in E(G)\}$$

and the neighbors of neighbors of x by

$$N_2(x) = \bigcup_{y \in N(x)} N(y) \setminus \{x\}.$$

- (a) Find out the distribution of the degree $|N(x)|$. Does it depend on the choice of x ?
- (b) Find out the expectation of $|N(x)|$.
- (c) Compute the probability of the event $|N(x) \cup N_2(x)| \leq 2$.

Continues on the next page...

6.4 *Facebook rumors.* Old schoolmates Aada, Bertta, and Cecilia form Facebook friendships so that each pair is connected with probability $p = 0.9$ independently of the others. If one of the three hears a rumor, she passes it immediately forward to all of her friends. If Aada hears a rumor, what is the probability that

- (a) Cecilia hears it?
- (b) Cecilia hears it, given that Aada and Bertta are not Facebook friends?
- (c) Cecilia hears it, given that she is not a Facebook friend of Aada?

6.5 *Random intersection graph.* Let $V = \{v_1, \dots, v_n\}$ and $W = \{w_1, \dots, w_m\}$ be disjoint sets. A random bipartite graph B with node set $V \cup W$ is defined as follows: each pair of nodes vw , where $v \in V$ and $w \in W$, is connected with probability r , independently of the other node pairs. The random intersection graph associated with the bipartite graph B is the graph $G = (V(G), E(G))$ where $V(G) = V$ and

$$E(G) = \{v_i v_j \in V^{(2)} : v_i w \in E(B) \text{ and } v_j w \in E(B) \text{ for some } w \in W\}.$$

- (a) Compute the probability that the node pair $v_1 v_2$ is connected in the random intersection graph G
- (b) Compute the expectation of the degree of node v_1 in the random bipartite graph B .
- (c) Compute the expectation of the degree of node v_1 in the random intersection graph G .