3.1 *Symmetric MC* [LPW08, Ex. 1.7]. A transition matrix P is symmetric if $P(x,y) = P(y,x)$ for all x, y. Show that if P is symmetric, then the uniform distribution on Ω is stationary for P.

3.2 *Reversible two-step chain* [LPW08, Ex. 1.8]. Let P be a transition matrix which is reversible with respect to the probability distribution π on Ω. Show that the transition matrix P^2 corresponding to two steps of the chain is also reversible with respect to π.

3.3 *Random king on the chessboard.* [Häg02, Ex. 6.1]. Consider a chessboard with a lone white king making random moves, meaning that at each move, he picks one of the possible squares to move to, uniformly at random.

(a) Prove that the corresponding MC is irreducible.

(b) Compute the unique stationary distribution π of the MC.
Hint: This Markov chain is reversible.

3.4 *Absorption time of a one-dimensional random walk* [LPW08, Ex. 2.3]. Consider a random walk on the path $\{0, 1, \ldots, n\}$ in which the walk moves left or right with equal probability except when at n and 0. At n, it remains at n with probability 1/2 and moves to $n-1$ with probability 1/2, and once the walk hits 0, it remains there forever. Compute the expected time of the walk's absorption at state 0, given that it starts at state n.

3.5 *Mean hitting time into a set.* [LPW08, Ex. 1.15]. For a subset $A \subset \Omega$, define $f(x) = \mathbf{E}_x(\tau_A)$, where $\tau_A = \min\{t \geq 0 : X_t \in A\}$ is the hitting time into A for a finite Markov chain (X_t) with transition matrix P. Show that

(a)
$$f(x) = 0 \quad \text{for } x \in A. \tag{1}$$

(b)
$$f(x) = 1 + \sum_{y \in \Omega} P(x,y) f(y) \quad \text{for } x \notin A. \tag{2}$$

(c) f is uniquely determined by (1) and (2).

References

[Häg02] Olle Häggström. *Finite Markov chains and Algorithmic Applications.* Cambridge University Press, 2002.

[LPW08] David A. Levin, Yuval Peres, and Elizabeth L. Wilmer. *Markov Chains and Mixing Times.* American Mathematical Society, 2008.